Optimal Scheduling for the Replacement of Components Subject to Technological Obsolescence

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Abstract : Most maintenance policies assume that failed or used components are replaced with identical units. Actually, such a hypothesis neglects the possible obsolescence of the components. When a new, more reliable and less consuming technology becomes available, a decision has to be made as for the replacement strategy to be used: old-type components can all be immediately replaced, or new-type units can be introduced progressively, each time a corrective action is undertaken. Partly corrective, partly preventive policies can also be envisioned.

This work tackles this issue in the case of a series system. It provides, under given modeling assumptions, the fully analytical expression of the expected total cost induced by each possible strategy, as well as the optimal replacement policy, as a function of the problem parameters.

Keywords : technological obsolescence, preventive maintenance, cost-based optimization.

1 – Introduction

Many papers devoted to the optimization of preventive or corrective maintenance policies, assume that failed or used components are always replaced by identical items. Actually, most components are subject to technological obsolescence: new components may appear on the market with the same (or even higher) capabilities but smaller failure rates and/or lower energy consumption. Managers then face an important question: how to optimally schedule the replacement of old-type units by new-type ones? Is it worth preventively replacing still working, old-type components by new-type ones? This question was considered in [Borgonovo et al.] in the case of one single component subject to aging, and which can be either periodically maintained or replaced by a technologically more advanced unit.

The present work investigates this issue in the case of a system of components. It focuses on the particular case of a series system made of n identical and independent components with constant failure rates, which may be instantaneously replaced by new-type ones. The latter are assumed (without any loss of generality) to become available at time 0. No preventive replacement is performed at the beginning; a new-type component is introduced in the system only to replace an old-type unit that has failed. This goes on until $K - 1$ corrective actions of this kind have been performed ($1 \leq K \leq n$). When the K-*th* old-type unit fails, all the remaining old-type units (the failed unit and the other ones) are preventively replaced with new-type ones, which leads to the simultaneous replacement of $n - K + 1$ old-type components. This

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strategy will be referred to as *strategy K*. Note that strategy n corresponds to a purely corrective approach, since it simply consists in replacing failed units without any preventive replacement. As for strategy 1, all units are changed as soon as one has failed. Besides, we also consider the so-called "strategy 0", where all the old-type units are replaced with new-type ones as soon as they appear on the market, i.e. at time 0. The objective of our study is to look for the optimal strategy K_{Opt}, which minimizes a given cost function, which is defined later on, with respect to a certain mission time, say t.

Such a problem was actually propounded and studied in [Elmakis et al.]. Their work takes into account different costs for preventive and corrective maintenance, as well as a rate of discount. Yet the treatment they perform relies on some assumptions, which are not always acceptable. First, the computation of the cost function is made under a large mission time assumption. In other words, they assume that a sufficient time interval has elapsed and that all replacements to be done in strategy K have already taken place. This prevents the manager from getting a genuine estimation of the cost induced by a given strategy on short and medium mission times, especially for large values of K. Secondly, no economical dependence in the replacement costs is accounted for, i.e. the cost of performing m simultaneous replacements is taken proportional to m. Moreover, different approximations are made all along the calculation, such as substituting the random sojourn time of the system in a state with its mean, instead of using its full distribution. Finally, the optimal K is numerically computed.

In this paper, the assumption on the mission time is relaxed, while the exact distributions of the time to the next failure are used instead of their means in all analytical developments. The optimal value of K is analytically provided, depending on the mission time and on the problem parameters. Moreover, an economical dependence for the replacements has been introduced: each solicitation of the repair team entails a fixed cost, no matter what type and number of actions must be achieved. Furthermore, the rate of energy consumption of the new components is taken smaller than that of the old-type ones.

This paper is organized as follows. Section 2 presents the modeling assumptions of the problem and the notations used. Important preliminary results are then gathered in Section 3, before giving in Section 4 the expression of the expected total cost entailed by the application of each possible strategy. The next section deals with the determination of the optimal replacement policy, and is followed in Section 6 by some numerical cases illustrating the results. Due to the lack of space, only milestones of the demonstrations are given here. The proofs of the different results can be found in [Mercier, Labeau] and can be asked to the authors. Concluding remarks and perspectives close the paper.

2 – Assumptions - Notations

Let λ_1 and λ_2 be the failure rates of the old- and new-type components, respectively. We recall that both failure rates are assumed to be constant, i.e. aging is not taken into account for both kinds of units.

For $0 \leq K \leq n$, let $\mathrm{E}(C_K([0,t]))$ be the expected cost of exploitation of the system on interval $[0,t]$ when strategy K is used. As already noted, a rate of discount, say i_r, is taken into account and all the costs are computed at time 0.

The following costs are considered:

- Replacement costs:
    - replacement of a single component in case of failure at time u such that $0 \leq u \leq t$: $(r + c_f)(1 + i_r)^{-u}$, where r denotes the cost entailed by the solicitation of the maintenance team and c_f is the cost induced by the failure of a component, and hence by the unavailability of the system,
    - simultaneous preventive replacements of n components at time 0: $r + n\, c_p$, where c_p is the unit cost per preventive replacement,
    - simultaneous preventive replacements of j components and corrective replacement of one component at time u such that $0 \leq u \leq t$: $(r + j\, c_p + c_f)(1 + i_r)^{-u}$.
- Energy consumption costs:
    - energy consumption of j new-type units on $[u, u+du]$: $j\eta\ (1 + i_r)^{-u}du$, where η is the energy consumption rate of the new-type components in monetary values. This leads to the following cost on $[t_1, t_2]$ (included in $[0, t]$):

$$\int_{t_1}^{t_2} j\eta(1+i_r)^{-u}\,du = j\frac{\eta}{\ln(1+i_r)}\left((1+i_r)^{-t_1}-(1+i_r)^{-t_2}\right)$$

- energy consumption of j old-type units on $[t_1, t_2]$:

$$j\frac{\eta+v}{\ln(1+i_r)}\left((1+i_r)^{-t_1}-(1+i_r)^{-t_2}\right)$$

with $v \geq 0$.
Note that the energy consumption rate of the new-type components (η) is assumed to be lower than for the old-type ones ($\eta + v$).

When strategy n is used (i.e. no preventive replacement is undertaken), we denote by $0 < T_1 < T_2 < \ldots < T_n$ the failure times of the old-type units. When strategy K is used (with $K \geq 2$), only failed old-type units are replaced at times $T_1, T_2, \ldots T_{K-1}$, whereas new-type components are substituted to all the $n - K + 1$ remaining old-type units at time T_K.

Finally, we will use the following notations:

$$a = \frac{r+c_f}{c_p} \quad ; \quad h = a\left(1-\frac{\lambda_2}{\lambda_1}\right)-1$$

$$b = hc_p + \frac{v}{\lambda_1} \quad ; \quad \alpha = \frac{n\lambda_1 + \ln(1+i_r)}{n\lambda_1}$$

3 – Preliminary Results

We summarize in this section some key results and definitions, which are needed in section 4 to calculate the total expected costs for the different possible strategies.

Lemma 1. *T_K admits f_K and F_K for probability density and cumulative distribution functions, respectively, with*

$$f_K(t) = \lambda_1.\, K C_n^K e^{-(n-K+1)\lambda_1 t}\left(1-e^{-\lambda_1 t}\right)^{K-1} \mathrm{I}_{[0,+\infty[}(t) \qquad , K=1\ldots n$$

and

$$F_{K+1}(t) = F_K(t) - C_n^K e^{-(n-K)\lambda_1 t}\left(1-e^{-\lambda_1 t}\right)^{K} \mathrm{I}_{[0,+\infty[}(t) \qquad , K=1\ldots n\text{-}1$$

Proof. The first result is easy to prove recursively using $T_K = (T_K - T_{K-1}) + \ldots + (T_2 - T_1) + T_1$ and the fact that $T_{i+1} - T_i$ follows an exponential distribution with rate $(n - i)\lambda_1$, $i=0\ldots n-1$ (with $T_0 = 0$). An integration by parts gives the second result. ■

Let Γ, B, and I_x be the gamma, beta and incomplete beta functions, respectively. We recall that:

$$\Gamma(v) = \int_0^{+\infty} e^{-x} x^{v-1}\,dx$$

$$B(v,w) = \int_0^1 u^{v-1}(1-u)^{w-1}\,du$$

$$I_x(v,w) = \frac{1}{B(v,w)}\int_0^x u^{v-1}(1-u)^{w-1}\,du$$

for all $v, w > 0$ and $0 \leq x \leq 1$.

Besides, we will use function R defined as:

$$R(t,v,w) = \lambda_1 \int_0^t e^{-\lambda_1 v u}\left(1-e^{-\lambda_1 u}\right)^{w-1}\,du$$

for all $t \geq 0$, $v, w > 0$.

Lemma 2. $\mathrm{E}\left(\mathrm{I}_{\{T_K \leq t\}}(1+i_r)^{-T_K}\right) = K C_n^K R(t, n\alpha - K + 1, K)$

where C_n^K is the binomial coefficient.

Proof. Due to the density function of T_K (see Lemma 1), this result is a direct consequence of the definition of R. ■

Lemma 3. *Let τ, $U_1, U_2, \ldots U_j, \ldots$ be random variables following exponential distributions with parameters μ' for τ and μ for all U_j, all independent with each other and independent of T_K. Then:*

$$\mathrm{E}\left(\sum_{j=1}^{+\infty} \mathrm{I}_{\{U_1+U_2+\ldots+U_j<t\}}(1+i_r)^{-(U_1+U_2+\ldots+U_j)}\right) = \mu \frac{1-(1+i_r)^{-t}}{\ln(1+i_r)}$$

$$\mathrm{E}\left(\sum_{j=1}^{+\infty} \mathrm{I}_{\{T_K+U_1+U_2+\ldots+U_j<t\}}(1+i_r)^{-(T_K+U_1+U_2+\ldots+U_j)}\right)$$
$$= \mu \frac{K C_n^K}{\ln(1+i_r)}\left[R(t, \alpha n-K+1, K)-(1+i_r)^{-t} R(t, n-K+1, K)\right]$$

$$\mathrm{E}\left(\sum_{j=1}^{+\infty} \mathrm{I}_{\{T_K+U_1+U_2+\ldots+U_j<\inf(t, T_K+\tau)\}}(1+i_r)^{-(T_K+U_1+U_2+\ldots+U_j)}\right)$$
$$= \mu \frac{K C_n^K}{\mu'+\ln(1+i_r)}\left[R(t, \alpha n-K+1, K)-e^{-\mu' t}(1+i_r)^{-t} R\left(t, n-K+1-\frac{\mu'}{\lambda}, K\right)\right]$$

Element of proof. Those results (a little more technical) may be proved using the fact that $U_1 + U_2 + \ldots + U_j$ has an Erlang distribution with parameters j and μ. ■

4 – The cost function

We here calculate the expected cost $\mathrm{E}(C_K([0,t]))$ induced by the application of strategy K on $[0,t]$. We start (Theorem 1) by expressing it in the case $K = 0$, what can be quite easily achieved based on Lemma 3. However, the approach adopted for strategy 0 is far from trivial when applied to strategy K, for $K \geq 1$, due to the change of composition of the system at each failure and at time T_K. As the composition of the system is the same before time T_K and after time T_{K+1}, no matter what strategy between K or $K+1$ is used, the analytical work is then facilitated by calculating first the cost difference between strategies K and K+1 (Lemma 4), and then by summing these cost differences to obtain the absolute expected cost for each possible strategy K (Theorem 2).

Theorem 1.

$$\mathrm{E}(C_0([0,t])) = nc_p + r + n(ac_p\lambda_2+\eta)\frac{1-(1+i_r)^{-t}}{\ln(1+i_r)}$$

Proof. When strategy *0* is used, the cost on [*0*, *t*] is due to the preventive replacement of *n* components at time *0*, to the corrective replacements of the new-type units (among *n*) that fail on [*0*, *t*] and to the energy consumption of the new-type units on [*0*, *t*]. Taking $\mu = n\lambda_2$ in Lemma 3, symbols $U_1, U_2, \ldots U_j, \ldots$ now represent the time intervals between successive failures on [*0*, *t*]. We get:

$$\begin{aligned}\mathrm{E}(C_0([0,t])) &= nc_p + r \\ &\quad + ac_p \mathrm{E}\left(\sum_{j=1}^{+\infty}\left(\begin{array}{l}\mathrm{I}_{\{U_1+U_2+\ldots+U_j\leq t<U_1+U_2+\ldots+U_{j+1}\}} \\ \left((1+i_r)^{-U_1}+(1+i_r)^{-(U_1+U_2)}+\ldots+(1+i_r)^{-(U_1+U_2+\ldots+U_j)}\right)\end{array}\right)\right) \\ &\quad + n\eta\frac{1-(1+i_r)^{-t}}{\ln(1+i_r)} \\ &= nc_p + r + ac_p \mathrm{E}\left(\sum_{j=1}^{+\infty}\mathrm{I}_{\{U_1+U_2+\ldots+U_j<t\}}(1+i_r)^{-(U_1+U_2+\ldots+U_j)}\right) + n\eta\frac{1-(1+i_r)^{-t}}{\ln(1+i_r)}\end{aligned}$$

which, after reduction, gives the result, using Lemma 3. ■

Lemma 4. *For $1 \leq K \leq n-1$:*

$$\mathrm{E}(C_{K+1}([0,t])-C_K([0,t]))=\frac{(K+1)\nu}{\lambda_1}C_n^{K+1}R(t,n\alpha-K,K+1)$$
$$+c_p\begin{bmatrix}(K+1)(n-K+h)C_n^{K+1}R(t,n\alpha-K,K+1)\\-(n-K)K\,C_n^K R(t,n\alpha-K+1,K)\end{bmatrix}$$

Proof. No matter what strategy between K or $K+1$ is used, the mixture of both types of components in the system is the same before time T_K. Consequently, when $t < T_K$, the difference of costs between these two strategies is null and we only have to consider the case $T_K \le t$. Besides, the composition of the system is the same after time T_{K+1} too, so that we only have to calculate the difference of costs on $[T_K, \min(t,T_{K+1})]$, which leads to split the case $T_K \le t$ between $T_K \le t < T_{K+1}$ and $T_{K+1} \le t$. We now get:

$$\begin{aligned}\mathrm{E}(C_{K+1}([0,t])-C_K([0,t]))=&\,\mathrm{E}\left((C_{K+1}([T_K,t])-C_K([T_K,t]))\mathrm{I}_{\{T_K\le t<T_{K+1}\}}\right)\\&+\mathrm{E}\left((C_{K+1}([T_K,T_{K+1}])-C_K([T_K,T_{K+1}]))\mathrm{I}_{\{T_{K+1}\le t\}}\right)\end{aligned}\tag{1}$$

and have to compute both of those terms. Actually, we only give hints here on how to deal with the *first* term of this expression, due to the reduced size of this paper.

When $T_K \le t < T_{K+1}$, the difference of costs between strategies K and $K+1$ on $[T_K, t]$ is due to:

- the n - K preventive replacements at time T_K in case of strategy K (to be added to the corrective replacement, which is common to both strategies),
- the corrective replacements on $[T_K, t]$, because of failures among the n (respectively K) new-type components for strategy K (respectively $K + 1$),
- the difference in energy consumption on $[T_K, t]$ between n - K old-type components in strategy $K + 1$, and the corresponding $n – K$ new-type units in strategy K.

Now, taking $\mu = n\lambda_2$ in Lemma 3, let $U_1, U_2, \ldots U_j, \ldots$ represent the time intervals between successive failures among n new-type components. In the same way, taking $\mu = K\lambda_2$, let $U'_1, U'_2, \ldots U'_j, \ldots$ represent the same time intervals for K new-type components. We now get:

$$\begin{aligned}&\mathrm{E}\left((C_{K+1}([T_K,t])-C_K([T_K,t]))\mathrm{I}_{\{T_K\le t<T_{K+1}\}}\right)\\
&=-(n-K)c_p\mathrm{E}\left((1+i_r)^{-T_K}\mathrm{I}_{\{T_K\le t<T_{K+1}\}}\right)\\
&-ac_p\mathrm{E}\left((1+i_r)^{-T_K}\sum_{j=1}^{+\infty}\begin{pmatrix}\mathrm{I}_{\{T_K+U_1+U_2+\ldots+U_j\le t<T_K+U_1+U_2+\ldots+U_{j+1}\}}\\\left((1+i_r)^{-U_1}+(1+i_r)^{-(U_1+U_2)}+\ldots+(1+i_r)^{-(U_1+U_2+\ldots+U_j)}\right)\end{pmatrix}\mathrm{I}_{\{T_K\le t<T_{K+1}\}}\right)\\
&+ac_p\mathrm{E}\left((1+i_r)^{-T_K}\sum_{j=1}^{+\infty}\begin{pmatrix}\mathrm{I}_{\{T_K+U'_1+U'_2+\ldots+U'_j\le t<T_K+U'_1+U'_2+\ldots+U'_{j+1}\}}\\\left((1+i_r)^{-U'_1}+(1+i_r)^{-(U'_1+U'_2)}+\ldots+(1+i_r)^{-(U'_1+U'_2+\ldots+U'_j)}\right)\end{pmatrix}\mathrm{I}_{\{T_K\le t<T_{K+1}\}}\right)\\
&+(n-K)\frac{\nu}{\ln(1+i_r)}\mathrm{E}\left(\left((1+i_r)^{-T_K}-(1+i_r)^{-t}\right)\mathrm{I}_{\{T_K\le t<T_{K+1}\}}\right)\end{aligned}$$

which reduces to:

$$\begin{aligned}&\mathrm{E}\left((C_{K+1}([T_K,t])-C_K([T_K,t]))\mathrm{I}_{\{T_K\le t<T_{K+1}\}}\right)\\
&=(n-K)\left(\frac{\nu}{\ln(1+i_r)}-c_p\right)\mathrm{E}\left((1+i_r)^{-T_K}\mathrm{I}_{\{T_K\le t<T_{K+1}\}}\right)\\
&-ac_p\mathrm{E}\left(\sum_{j=1}^{+\infty}\mathrm{I}_{\{T_K+U_1+U_2+\ldots+U_j\le t\}}(1+i_r)^{-(T_K+U_1+U_2+\ldots+U_j)}\mathrm{I}_{\{t<T_{K+1}\}}\right)\\
&+ac_p\mathrm{E}\left(\sum_{j=1}^{+\infty}\mathrm{I}_{\{T_K+U'_1+U'_2+\ldots+U'_j\le t\}}(1+i_r)^{-(T_K+U'_1+U'_2+\ldots+U'_j)}\mathrm{I}_{\{t<T_{K+1}\}}\right)\\
&+(n-K)\frac{\nu}{\ln(1+i_r)}(1+i_r)^{-t}(F_K(t)-F_{K+1}(t))\end{aligned}$$

A similar expression can be written for the second term of expression (1).

Adding both terms and using Lemmas 1, 2 and 3 then leads, after some tedious computations, to the expected result. ■

Theorem 2. *For* $1 \le K \le n-1$:

$$\mathrm{E}\left(C_K([0,t])\right) = n\left(ac_p\lambda_2+\eta\right)\frac{1-(1+i_r)^{-t}}{\ln(1+i_r)} + nc_p\, C_{n-1}^{K-1}(n-K)R(t, n\alpha-K+1, K) + \sum_{j=1}^{K}\left(c_p+b\right) jC_n^j R(t, n\alpha-j+1, j)$$

$$\mathrm{E}\left(C_n([0,t])\right) = n\left(ac_p\lambda_2+\eta\right)\frac{1-(1+i_r)^{-t}}{\ln(1+i_r)} + n\left(c_p+b\right)\frac{1-e^{-(n(\alpha-1)+1)\lambda_1 t}}{n(\alpha-1)+1}$$

Proof. We begin with computing $\mathrm{E}(C_1([0,t]))$ in the same way as $\mathrm{E}(C_0([0,t]))$ (with the help of Lemma 2 and 3). We then write $\mathrm{E}(C_K([0,t])) = \mathrm{E}(C_1([0,t])) + \sum_{j=1}^{K-1}\left(\mathrm{E}(C_{j+1}([0,t])) - \mathrm{E}(C_j([0,t]))\right)$ for $1 \le K \le n$ and conclude with Lemma 4.
In the case $K = n$, the expression can be compacted to the formula mentioned above, using a binomial expansion, after substituting R with its definition. ■

5 – The optimal strategy

This section provides the value of K_{Opt}, as a function of the mission time and of the problem parameters. The results are summed up in Theorem 3 below.

In the first case of a single component, a condition is given on the problem parameters, under which corrective replacement of the component is always optimal ($K_{Opt} = 1$), no matter what the mission time is. Under the opposite condition, it is shown that, if the mission time t is greater than some characteristic time t_1, it is then worthy to preventively replace the component at time 0 ($K_{Opt} = 0$). If the mission time is smaller than t_1, then the best policy is to perform only corrective replacements.

In the multi-component case, it is shown that only three strategies may be optimal, i.e. strategies 0, 1 and n; it is quite noticeable that strategies 2, 3, … or $n-1$ never are optimal. More precisely, as in the single-component case, under some condition on the problem parameters, it is shown that a purely corrective replacement policy is optimal, no matter what the mission time is. Under the opposite condition, if the mission time t is large (see Theorem 3 just below for details), the best solution is to preventively replace all components very quickly, either at time 0 (strategy 0) or at the first failure of one component of the system (strategy 1). Here again, if the mission time is small, then the best is still to perform only corrective replacements, as intuitively expected.

Theorem 3.

For $n = 1$:

1. *If* $b \le \alpha r + (\alpha-1)c_p$: $K_{Opt} = 1$
2. *If* $b > \alpha r + (\alpha-1)c_p$: *let* $t_1 = \dfrac{1}{\lambda_1\alpha}\ln\left(\dfrac{c_p+b}{b-(\alpha-1)c_p-\alpha r}\right)$

   For $t \le t_1$: $K_{Opt} = 1$ *and for* $t > t_1$: $K_{Opt} = 0$

For $n \ge 2$:

1. *If* $b \le (\alpha-1)nc_p$: $K_{Opt} = n$
2. *If* $b > (\alpha-1)nc_p$: *let* t_0 *be the single* t *such that*

$$n\left(c_p+b\right)\frac{1-e^{-(n(\alpha-1)+1)\lambda_1 t}}{n(\alpha-1)+1}-\left(nc_p+b\right)\frac{1-e^{-n\alpha\lambda_1 t}}{\alpha}=0$$

a. *If* $b \le \alpha r+(\alpha-1)nc_p$ *: for* $t \le t_0$ *:* $K_{Opt}=n$ *and for* $t>t_0$ *:* $K_{Opt}=1$.

b. *If* $b>\alpha r+(\alpha-1)nc_p$ *:*

Let $t_1=\frac{1}{n\lambda_1\alpha}\ln\left(\frac{nc_p+b}{b-n(\alpha-1)c_p-\alpha r}\right)$ *and* $t_2=\frac{1}{(n(\alpha-1)+1)\lambda_1}\ln\left(\frac{n(c_p+b)}{nb-r-n(nc_p+r)(\alpha-1)}\right)$

i. *If* $t_2<t_0$ *(or* $t_1<t_0$*): for* $t\le t_2$ *:* $K_{Opt}=n$ *and for* $t>t_2$ *:* $K_{Opt}=0$.

ii. *If* $t_2\ge t_0$ *(or* $t_1\ge t_0$*): for* $t\le t_0$ *:* $K_{Opt}=n$*, for* $t_0\le t\le t_1$ *:* $K_{Opt}=1$ *and for* $t>t_1$ *:* $K_{Opt}=0$.

Elements of proof. We first show that the best strategy among 1, 2, … and *n* is strategy 1 or *n*. As a consequence, the optimal strategy has to be searched among 0, 1 and *n*. We then compare those three strategies two by two and finally conclude. The proof is quite lengthy and can be found in [Mercier, Labeau]. ■

6 – Examples

All the computations of this section have been made with Matlab. We recall that the incomplete beta function, the gamma function and its logarithm (gammaln) are implemented in Matlab. Expressions with products or ratios of gamma functions have been computed with gammaln instead of gamma to prevent overflow (using gamma(x) = exp(gammaln(x))).

For each example, two different time windows have been provided for the same figure.

6.1 – Example 1:

Let

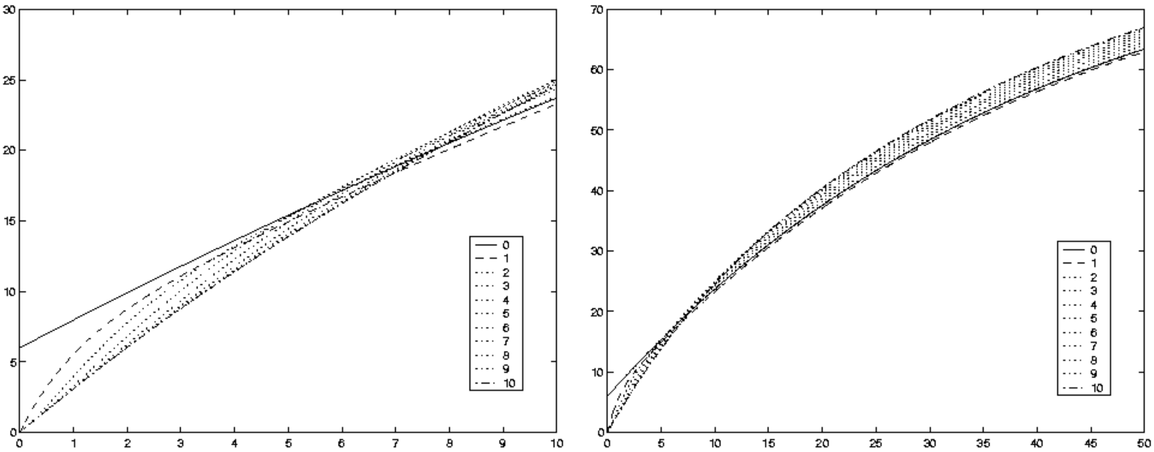
$$n=10,\ c_f=1,\ c_p=0.5,\ r=1,\lambda_1=0.1,\ \lambda_2=0.05,\ i_r=0.025,\eta=0.1,\nu=0.02$$

We derive:

$$b-(\alpha-1)nc_p\cong 0.5765>0$$

$$b-\alpha r-(\alpha-1)nc_p\cong -0.4482<0$$

This situation corresponds to the case 2.a. of theorem 3.



Figures 1 and 2: $\mathrm{E}\left(C_K\left([0,t]\right)\right)$ *with respect of t for K from 0 to 10, Example 1.*

The time evolution of the expected cost associated with each strategy is presented in Figures 1 and 2. We check that $K_{Opt}=10$ for $t\le t_0\cong 6.9101$, and $K_{Opt}=1$ for $t>t_0$.

6.2 – Example 2:

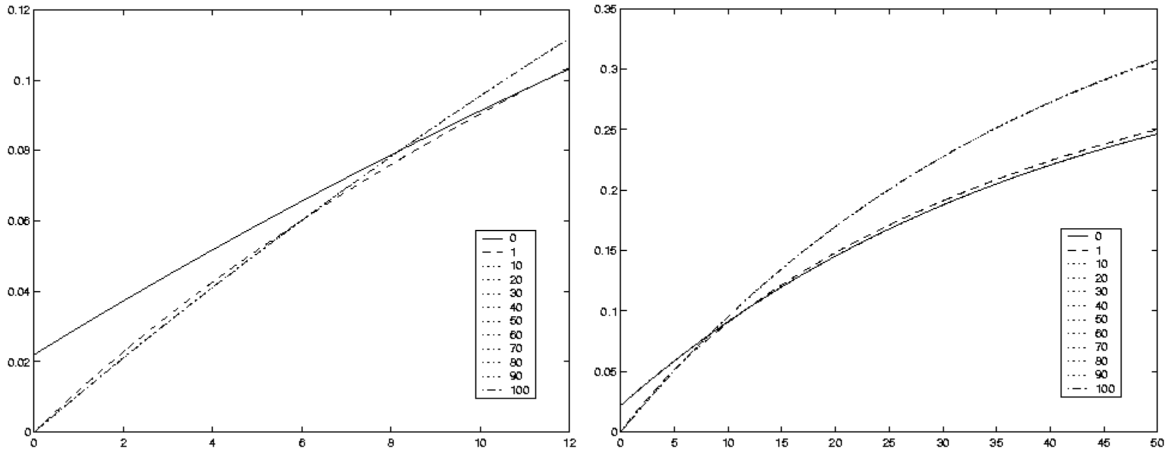
Let

$$n = 100,\ c_f = 0.05,\ c_p = 0.0001,\ r = 0.012, \lambda_1 = 0.0015,$$
$$\lambda_2 = 0.0011,\ i_r = 0.025, \eta = 0.00001, \nu = 0.000005$$

We derive:

$$b - (\alpha - 1) n c_p \cong 0.0181 > 0$$
$$b - \alpha r - (\alpha - 1) n c_p \cong 0.0041 > 0$$
$$t_0 \cong 6.0534 < t_1 \cong 11.2853 \ (and\ t_0 < t_2 \cong 8.2041)$$

Case 2.b.ii. of theorem 3 has thus to be considered this time.



Figures 3 and 4: $\mathrm{E}(C_K([0,t]))$ *with respect of t for K = 0, 1, 10, 20, … 100, Example 2.*

We can check in Figures 3 and 4 that for $t \le t_0$*: K_{Opt} = 100, for* $t_0 < t \le t_1$*: K_{Opt} = 1 and for* $t > t_1$*: K_{Opt}=.0.*

7 – Conclusions

This paper has provided the analytical treatment of a simple case of an important industrial issue, i.e. the optimization of the replacement strategy of components subject to technological obsolescence. Analytical expressions could be obtained assuming a.o. a series system, no aging of both old-type and new-type units, no common cause failures, instantaneous replacements and a full compatibility of the new components.

Future work should give allowance to more realistic situations in which these hypotheses are relaxed. Issues to be tackled include a.o. problems of compatibility between both types of components, common cause failures, more complex system structures, and so on. This implies soon to give up the analytical work and to resort to Monte Carlo simulation. Yet these analytical solutions for the particular case given in this paper will turn out to be useful in validating the simulation approach.

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