

# Monte Carlo Optimization of the Replacement Strategy of Components subject to Technological Obsolescence

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## Abstract

Components are technologically obsolescent when challenger units with higher performances become available. An optimal strategy mixing corrective and preventive replacements must be defined in order to minimize the expected total cost induced by this change of component generation. Realistic operational assumptions can be accounted for, as costs are estimated by a Monte Carlo simulation.

## 1 Introduction

Many papers devoted to the optimization of preventive or corrective replacement policies assume that, when components fail, identical items are always available to perform their replacement. Actually, most components are subject to technological obsolescence [1]: new components may appear on the market with the same function and the same (or even higher) performances but smaller failure rates. Managers then face the following issues: how to optimally schedule the replacement of old-type units by new-type ones? When is it worth preventively replacing still working, old-type components by new-type ones, in order to balance optimally the replacement costs and the subsequently expected reduced maintenance expenses?

The present work investigates these issues in the particular case of  $n$  identical components. The replacement strategy considered in this paper rests on a two-stage procedure [2]: first new-type components are used only to replace failed old-type units; then, after  $K$  corrective actions of this kind, all remaining old-type units are replaced by new-type ones. Two particular cases of this  $K$ -strategy correspond to:

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- $K=0$ , where all the components are preventively replaced by new-type ones as soon as they become available, and to
- $K=n$ , where no preventive action is undertaken.

The optimal  $K$  minimizing the total cost function must be obtained.

A full analytical treatment for a simplified problem of this kind is given in [3]. We will briefly explain this analytical solution and its underlying hypotheses in section 2. Such a treatment provides us with a reference solution for a Monte Carlo (MC) approach of more complicated replacement problems under obsolescence conditions, which are the subject of this work. Section 3 contains a description of the model considered in this paper and based on more realistic assumptions than the ones used to obtain the analytical solution. In section 4, practical aspects of the implementation of the MC simulation are discussed, while section 5 illustrates some aspects of the whole model on some numerical results. Finally, we conclude by suggesting some possible perspectives and extensions of this model.

## 2 An analytical test-case

We describe here shortly a simplified problem [3] consisting of a series system made of  $n$  identical and independent units with constant failure rates  $\lambda$  and  $\mu$  for old- and new-type units, respectively.

The authors considered a discount rate  $i_r$  and the following cost structure: a fixed cost  $r$  per intervention, a failure cost  $c_f$  per corrective replacement and a cost  $c_p$  per preventive replacement. Discounting the costs at time 0, their expression becomes:

- Cost of a corrective replacement at time  $u$ :  $(r + c_f) (1 + i_r)^{-u}$
- Cost of  $(n - K)$  simultaneous preventive replacements and one corrective replacement at time  $u$ :  $(r + (n - K) c_p + c_f) (1 + i_r)^{-u}$

As for energy consumption, new-type units have a smaller consumption rate than the old-type ones ( $\eta$  and  $(\eta + \nu)$ , respectively).

With these assumptions, an analytical solution can be obtained; it was shown that the only values of  $K$  minimizing the mean total cost are either 0, 1 or  $n$ , no matter what values the system parameters may take. Whether these conclusions keep valid or not when making the model more realistic is one of the goals of this work.

## 3 Description of the model

The more realistic model that has been developed gives allowance to additional features, which are described here below.

The system we chose to study is made of  $n$  identical components, expected to produce simultaneously. This is an alternative description to the series system mentioned in the previous section. The  $n$  units are now dependent. Indeed, common mode failures are accounted for and modeled through the Multiple Greek Letters scheme [4], for which a set of  $n-1$  conditional probabilities  $\beta_i$ , such that

$$\beta_i = P(\text{at least } i+1 \text{ failures occur in a common mode failure} \mid \text{at least } i \text{ failures occur in this common mode failure}),$$

must then be provided. The risk of common mode failures should thus foster an faster preventive transition to the new, more reliable equipments.

Component aging is introduced for both old- and new-type units, their failure times being distributed according to Weibull laws. A single repairman is considered, and both corrective and preventive replacement times are modeled by Erlang distributions. Hourly costs are associated to these non-zero replacement times in three different cases: manpower costs, lack of production during the failure and replacement period, and manpower costs when replacement times exceed a maximum duration (e.g. a normal working day).

The addition of the latter category is more critical since, as common mode failures and finite replacement times can lead to multiple simultaneously failed equipments, replacements are pursued within the same intervention until all failed components are replaced. If the condition to perform the  $K$ -strategy is fulfilled at this time (i.e. replacement of at least  $K$  old-type components since the beginning of the study), the intervention of the repairman is continued until the  $K$ -strategy is fully applied, even if additional failures occur meanwhile. It should also be underlined that the priority is always given to corrective maintenance actions over preventive ones, in order to decrease the costs. This means that the application of the  $K$ -strategy will immediately be postponed, if any new failure is detected, once the current preventive replacement is completed. Another consequence of this priority rule concerns the starting point of the massive preventive replacements, which will not always correspond to the exact number  $K$  of old-type components replaced by challenger ones, if other old-type units are still failed once this limit  $K$  is reached.

We also introduce the concept of incompatibility between the component generations. The on-site implementation of new-type components could turn out to be problematic, and some replacements could not be immediately successful, as technicians are not familiar yet with this new technology. Information on the installation procedure and experience should increase with time, however, thereby reducing the risk of incompatibility. In order to account for this phenomenon in a simple way, we consider for the new-type components a non-zero probability  $p_{inc}$  to fail on demand when started after replacement. This probability is made of two contributions: a probability  $p_s$  that the new component does not start when solicited, and a probability of incompatibility. The latter remains constant all along a series of replacements without interruption. It is decreased by a constant factor  $f_{red}$  from one intervention to the next one, to take into consideration the additional available information and increased experience. We can thus write:

$$p_{inc} = p_s + \frac{p_o}{(f_{red})^{n_{int}}}$$

where  $n_{int}$  is the number of interventions on the system. This incompatibility should not favor early preventive replacements.

## 4 Monte Carlo simulation

Considering the assumptions of our model, a MC treatment has to be envisioned. Our algorithm presents two major parts, for the respective treatment of failures and replacements. Once the units involved in a possible common mode failure are determined, replacement times are successively sampled from an Erlang law for each of them. The replacements are always achieved in the order of failure occurrences, given the repairman availability. A check for failures occurring during the replacement time of a unit is made after each replacement, as an intervention is not interrupted as long as failed components are still to be replaced.

During the algorithm development, attention needs to be paid to the interaction between a direct simulation, with independent samplings of the component failure times, and common mode failures that introduce a strong dependence between all units of the same type, thus at the system level. For instance, failures occurring during a replacement period cannot concern the component that is being replaced; as these “secondary” failures are checked at the end of the replacement time, we had to store the component that was just replaced in a stack, before sampling possible common cause failures.

## 5 Results

Our MC code was first validated on the simplified problem with an analytical solution presented in section 2. From this point on, simulations were performed in more realistic situations. Yet results obtained on the whole model are not easily interpreted. We thus chose first to activate each option separately, and to study how it affects the relative behavior of the different strategies that was observed in the analytical test-case. Due to the lack of space, we can only briefly present here a limited sample of the possible results.

As a first illustration, figure 1 shows the mean total cost as a function of time and for different values of  $K$ , considering a risk of incompatibility between the component generations. It can be observed that the conclusions of the analytical test-case are no longer valid, i.e. that strategies other than  $K = 0, 1$  or  $n$  can be optimal, as it is the case for strategy  $K = 7$  on part of the displayed time interval. This conclusion is reinforced by figure 2, which displays the expected cost induced by all the possible strategies, up to two different epochs. Though the differences are limited in magnitude, it can be seen that strategy  $K = 4$  is optimal when applied on the time interval  $[0,20]$ , while the optimum goes to strategy  $K = 3$  on  $[0,25]$ .

In a second example, we consider the effect of non-zero replacement times with an hourly failure penalty, but without manpower costs. Figure 3 compares, for  $K = 1$  and 10, the expected total cost obtained with our MC code (i.e. a single repairman) with the sum of the analytical cost and the increase in cost obtained in the assumptions of this second example, for a set of independent components (i.e. with  $n$  repairmen). As replacement times for the different components may overlap, two trends in opposite directions affect the MC results: a cost reduction due to less repetitions of the cost per intervention  $r$ , and a negative effect caused by possibly

longer sojourns in the failure state while waiting for the only repairman to be available again. This second effect obviously overcomes the expected cost reduction for the choice of parameters made. Such a difference increases when long series of preventive replacements are to be completed, i.e. for small values of  $K$ .

## 6 Conclusions

Though technological obsolescence affects most components of industrial installations, the replacement strategy of old-type units by challenger equipments with higher performances has often been neglected in the planning of preventive maintenance actions. This work dealt with replacement policies mixing both corrective and preventive interventions. It presented the first results of a MC cost-based optimization of such replacement policies for  $n$  identical units, with realistic operational rules and maintenance constraints, which make impossible an analytical treatment of the problem. Preliminary results show the impact of each modeling assumption on the expected cost associated to the different strategies considered, and to which extent they affect the choice of an optimal policy.

Future work will first concentrate on determining regions of the parameter space where specific strategies are optimal. Additional aspects will then be incorporated in the problem, such as e.g. redundancy of equipments, spare parts inventory ...

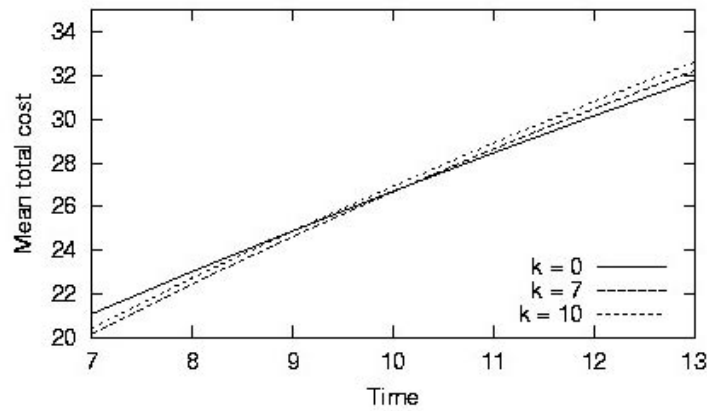


Figure 1. Mean total cost in presence of incompatibility probability with  $n = 10$ ,  $r = 1$ ,  $c_f = 1$ ,  $c_p = 0.5$ ,  $\lambda = 0.05$ ,  $\mu = 0.025$ ,  $\eta = 0.02$ ,  $v = 0.1$ ,  $i_r = 0.025$ ,  $p_s = 0$ ,  $p_o = 0.3$ ,  $f_{red} = 1.2$

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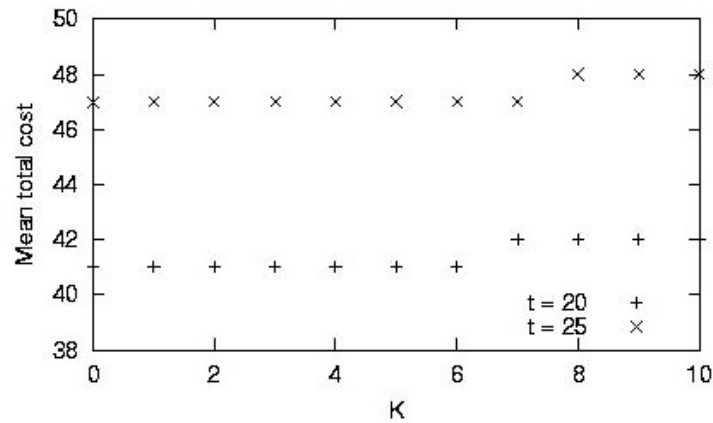


Figure 2. Mean total cost at two epochs as a function of  $K$

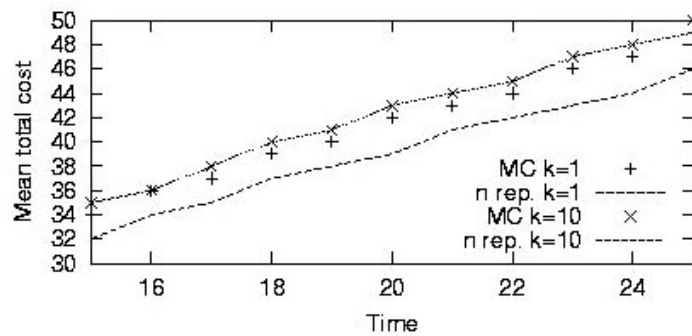


Figure 3. Effect of the number of repairs in case of non-instantaneous replacements Erlang distribution of order 2 with  $\rho = 10$ , hourly failure cost = 1.5