Maintenance effect modelling and optimization of a two-components system

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ABSTRACT: This article deals with the preventive maintenance optimization of a two-components system used at the SNCF (French National Railway Society). Both components have two failure modes and the system functioning mode makes the components dependent. This system is presently submitted to a periodic preventive maintenance policy. The aim of this paper is to study the eventual benefits provided by some adjustments on this periodic policy. By a preventive maintenance action, failed components are presently renewed and working components are adjusted. A slight modification of a first-order Arithmetic Reduction of Age (ARA_1) model is used to describe the components adjustments, together with Bertholon distributions for the intrinsic components life-times. Maximum likelihood estimates are computed for the model parameters. A Piecewise Deterministic Markov Process is used to model the system behavior both under the present preventive maintenance policy and under the adjusted ones. Several reliability indicators are finally numerically assessed and optimized for the different maintenance policies, which allows to better see their advantages and drawbacks.

1 INTRODUCTION

For a railway infrastructure like SNCF (French National Railway Society), maintenance of the infrastructure is a major task because a failure causes delays and client dissatisfaction. Moreover, failures increase maintenance cost. The SNCF has hence initiated research in order to model the involved systems, in view of some improvement in their preventive maintenance. This article deals with a two-components system. The two components are functionally dependent. Those systems are used by the SNCF in great number and we want to optimize their maintenance given that they have already worked for a while. The nature of the system is not revealed because of confidentiality issue.

To ensure the proper functioning of the system, a preventive maintenance action is annually undertaken. During a preventive maintenance action, the SNCF agent replaces the broken components if any and adjusts the working components. In order to model the adjustments effect on the two components, we use a virtual age model: the adjustments effectiveness is modelled by an ARA_1 (first-order Arithmetic Reduction of Age) model (Doyen and Gaudoin 2004). Intrinsic failure rate is of Weibull type. We introduce a modification of the ARA_1 model using a Bertholon type intrinsic failure rate (Bertholon 2001). This new model is called first-order Arithmetic Reduction of Age with Bertholon Adaptation ($ARABA_1$).

Because of the components aging, usual Markov jump processes with finite state space cannot be used. Consequently, in order to model the system, Piecewise Deterministic Markov Processes (PDMP) are used. Those processes are described in (Davis 1984). Their numerical assessment is often established by Monte Carlo simulations (Zhang et al. 2008); however this method usually takes too much time to optimize maintenance. An alternate method is here used: the quantities of interest can be expressed using the PDMP marginal distributions, which are known to be solutions of a set of partial differential equations called Chapman-Kolmogorov equations. A finite volume scheme, which is an explicit version of the algorithm presented in (Eymard et al. 2008) in a simplified framework, provides numerical estimates for the PDMP marginal distributions, as solution of this scheme.

This paper is organized as follows: in Section 2, the two-components system is presented. In Section 3, the models used to estimate the components life-time (Weibull and Bertholon) and the maintenance effect (ARA_1 and $ARABA_1$) are presented as well as the estimation results. In Section 4, we present the PDMP used to model the maintained system. In Section 5, a new preventive maintenance strategy with preventive renewal of components is presented and modelled with PDMPs. The associated cost functions are provided, with respect of the PDMPs marginal distributions. An optimal maintenance strategy which minimizes the cost function is determined. Conclusive remarks end this paper in Section 6.

2 THE TWO-COMPONENTS SYSTEM

2.1 *System presentation*

The system has two components, A and B, which are functionally dependent. Both of them can fail in two failure modes denoted by F_1 and F_2. When a component fails in failure mode F_1 and the other one is still working, the system works fine. Conversely, when a component fails in failure mode F_2, the system does not work anymore regardless of the other component status. Such a failure leads to a corrective maintenance action; broken components are instantly replaced by new ones. When the two components fail in failure mode F_1 one after the other, not only the system no longer works but it creates an undesirable event as well. A corrective maintenance action is undertaken and the components are instantly replaced by new ones. The system failures can be classified into two categories, the first being a classic failure and the other a more severe failure. The two of them will be quantified.

2.2 *Preventive maintenance strategy*

A system failure leads to a corrective maintenance action. In order to avoid the undesirable event, the system is also preventively maintained. A SNCF agent is sent to the system each year. During a preventive maintenance action, the agent replaces the broken components if any and adjusts the working components.

Both corrective and preventive maintenance actions are considered as instantaneous because the duration of a maintenance action is negligible compared to the life-time of components.

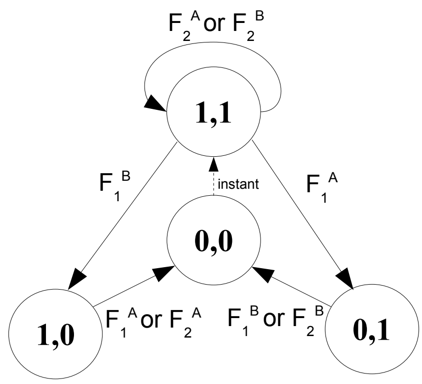

Figure 1: Markov graph of PDMP discrete states

Figure 1 represents possible transitions between system states with 1 for up and 0 for down. System does not stay in state $(0, 0)$ because corrective maintenance actions are instantaneous.

In order to quantify the effect of the preventive maintenance actions on the components life-time, a virtual age model is used. It is presented in next section.

3 COMPONENTS LIFE-TIME AND MAINTENANCE EFFECT MODELLING

3.1 *Model presentation*

(Doyen and Gaudoin 2004) propose Arithmetic Reduction of Age (ARA) models in order to quantify maintenance actions effect on a component life-time. The principle of such models is to introduce a virtual age for a system under maintenance, which is reduced at each maintenance time. A first-order ARA model is here used (ARA_1), which reduces the components virtual age at each maintenance time by a fraction ρ of the elapsed time since the last maintenance action. Parameter ρ measures the maintenance efficiency. Let $\lambda_\theta(t)$ be the component intrinsic failure rate (failure rate of the unmaintained component) with θ the intrinsic failure rate parameters.

An ARA_1 model is defined by its failure intensity :

$$\lambda_t^{ARA_1} = \lambda_\theta \left(t - \rho T_{N_{t^-}} \right) \qquad (1)$$

with T_i the i^{th} maintenance action time and N_{t^-} the number of maintenance actions occurred before time t. The ρ coefficient models maintenance effect and $\rho \in [0, 1]$. Depending on the values of ρ, we have different maintenance effects:

- $\rho = 1$: As Good As New (AGAN),
- $\rho = 0$: As Bad As Old (ABAO),
- $\rho \in]0; 1[$: maintenance is effective,
- $\rho < 0$: maintenance is damaging.

Let us consider our case of application. ARA_1 model is used to quantify the effect of the preventive maintenance actions on the components life-time. A data base providing information on corrective and preventive maintenance actions of n components is at our disposal, we set for $i \in \{1, \cdots, n\}$:

- t_i: minimum between life-time of component i and censoring time,
- δ_i: nature of data t_i. If t_i is a life-time $\delta_i = 1$, if t_i is a censoring time $\delta_i = 0$,
- T_k^i: time of the k^{th} preventive maintenance action undertaken on component i which occurs every year,
- $N_{t_i^-}$: number of preventive maintenance actions occurred on component i before t_i.

The ARA_1 likelihood function is, see (Doyen and Gaudoin 2004):

$$L\left(t_i, \delta_i, T_1^i, \cdots, T^i_{N_{t_i^-}}, i \in \{1, \cdots, n\}, \theta, \rho\right) =$$

$$\prod_{i=1}^{n} \left[\lambda_\theta\left(t_i - \rho T^i_{N_{t_i^-}}\right)\right]^{\delta_i}$$

$$exp\left(-\sum_{k=0}^{N_{t_i^-}-1}\left(\int_{T_k^i}^{T_{k+1}^i} \lambda_\theta\left(t - \rho T_k^i\right) dt\right)\right)$$

$$exp\left(-\int_{T^i_{N_{t_i^-}}}^{t_i} \lambda_\theta\left(t - \rho T^i_{N_{t_i^-}}\right) dt\right) \quad (2)$$

First, the intrinsic failure rate is supposed to be of the Weibull type. The Weibull failure rate is:

$$\lambda_\theta(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \quad (3)$$

with $\theta = (\eta, \beta)$. The ARA_1 model failure intensity associated with Weibull distribution is:

$$\lambda_t^{ARA_1} = \frac{\beta}{\eta}\left(\frac{t - \rho T_{N_{t-}}}{\eta}\right)^{\beta-1} \quad (4)$$

Components failure intensity associated with ARA_1 model and Weibull hypothesis does not fit with the non-parametric failure rate, see Figures 2 and 3. Non-parametric failure rate does not seem to start at zero and seem to be constant during the first years of functioning. This phenomenon can be modeled with Bertholon distribution.

We now suppose that the intrinsic failure rate is of the Bertholon type. The Bertholon distribution models a constant failure rate until time t_0 and an increasing failure rate after. The first part corresponds to an Exponential distribution and the second part corresponds to the minimum between an Exponential distribution and a Weibull distribution. The Bertholon failure rate is :

$$\lambda_\theta(t) = \frac{1}{\eta_0} + \frac{\beta}{\eta}\left(\frac{(t - t_0)^+}{\eta}\right)^{\beta-1} \quad (5)$$

with $\theta = (\eta_0, t_0, \eta, \beta)$. In order to model a preventive maintenance action efficiency for a Bertholon intrinsic failure rate, we choose to modify the ARA_1 model. A maintenance action occurred before time t_0 is supposed to have no effect on the component life-time. The effect is only modelled after time t_0. This assumption is justified by the fact that before t_0, failures are assumed to be accidental and maintenance action cannot prevent them. This new model is called first-order Arithmetic Reduction of Age with Bertholon Adaptation $ARABA_1$. The $ARABA_1$ model failure intensity is expressed by the following:

$$\lambda_t^{ARABA_1} = \frac{1}{\eta_0} + \frac{\beta}{\eta}\left(\frac{(t - t_0)^+ - \rho\left(T_{N_{t-}} - t_0\right)^+}{\eta}\right)^{\beta-1} \quad (6)$$

In order to estimate the model parameters, we use the Maximum Likelihood Estimation (MLE) method. The likelihood function is maximized with a Simulated Annealing algorithm.

3.2 *Results*

Results are confidential so legends are hidden. Figures 2 and 3 represent components A and B failure rates estimated with different methods : non-parametric method (Kaplan-Meier), ARA_1 associated with Weibull distribution and $ARABA_1$. We can observe that in the two cases, failure rates estimated with $ARABA_1$ are closer to non-parametric estimates than failure rate estimated with ARA_1 associated with Weibull distribution. In the following, we consequently use $ARABA_1$ results.

Using these results, we model the maintained system with a PDMP. The model is presented in next section.

4 SYSTEM MODELLING

4.1 *Piecewise Deterministic Markov Processes*

PDMPs have been introduced by Davis in 1984 in (Davis 1984). This type of modeling has been used by Devooght (Devooght 1997) for nuclear issues. A PDMP is a hybrid process $(I_t, X_t)_{t\geq 0}$. The first component I_t is discrete, with values in a finite state space

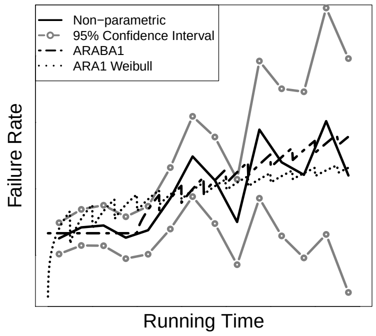

Figure 2: Component A failure rate

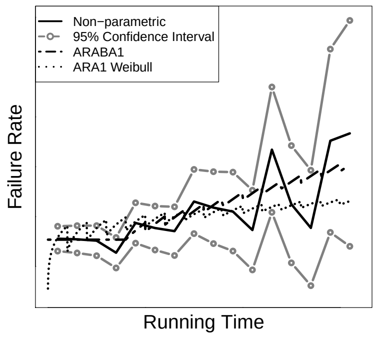

Figure 3: Component B failure rate

E. Typically, it indicates the state - up or down - for each component of the system at time t. The second component X_t, with values in a Borel subset $G \subset \mathbf{R}^d$, stands for environmental conditions, in our case, the components entry into service dates and the time t; the time unit is year. This means that a PDMP can model a system with aging components. The two parts I_t and X_t interact one with each other: the process jumps at countably many isolated random times; by a jump from $(I_{t^-}, X_{t^-}) = (i, x)$ to $(I_t, X_t) = (j, y)$ (with $(i, x), (j, y) \in E \times G$), the transition rate between the discrete states i and j depends on the environmental condition x just before the jump, and is a function $x \to a(i, j, x)$. Similarly, the environmental condition X_t just after the jump, is distributed according to some distribution $\mu_{(i,j,x)}(dy)$, which depends on both components just before the jump (i, x) and on the after jump discrete state j. So the transition kernel which governs the transition between (i, x) and (j, y) is:

$$b((i,x),(j,dy)) = a(i,j,x)\mu_{(i,j,x)}(dy) \tag{7}$$

Between jumps, the discrete component I_t is constant and the evolution of the environmental condition X_t is deterministic, solution of a set of differential equations which depends on the fixed discrete state: given that $I_t = i$ between two jumps, X_t is solution of

$$\frac{dy}{dt} = v(i,y) \tag{8}$$

In order to model a PDMP jump occurring at a deterministic time, an after-jump distribution may be defined and is denoted by $q((i,x),(j,dy))$ with $x = (x_1, x_2, \cdots, x_{d-1}, k)$ and k a deterministic jump time. In our case such a distribution models a preventive maintenance action which periodically occurs every year so $k \in \mathbf{N}^*$.

The two-components maintained system can be modelled with a PDMP with state space $E = \{(1,1),(1,0),(0,1)\}$ (1 for up and 0 for down). The state $(0,0)$ is never reached because maintenance actions are instantaneous, so that the system runs continuously. Components failure rates depend on their age so the environmental variable have to contain this information. Be x_A the date of entry into service of component A and x_B the date of entry into service of component B, the PDMP environmental variable is (x_A, x_B, t) so the space dimension d is 3. Let T be the time horizon and $G = [0,T]^3$. Between two jumps, only environmental variable t evolves at speed 1 so that we have, for all $i \in E$:

$$v(i,(x_A,x_B,t)) = (0,0,1) \tag{9}$$

In order to identify each component failure mode, we define the probabilities $p^A_{F_i}$ and $p^B_{F_i}$ with $i \in \{1,2\}$, where $p^A_{F_i}$ (respectively $p^B_{F_i}$) is the probability that component A (respectively component B) fails in mode F_i given that it fails. We have for $C \in \{A,B\}$

$$p^C_{F_1} + p^C_{F_2} = 1 \tag{10}$$

Let $(\eta^C_0, t^C_0, \eta^C, \beta^C)$ be Bertholon coefficients and ρ_C maintenance effect coefficient of component C with $C \in \{A,B\}$. At time t, maintained components A and B failure rates are, with $C \in \{A,B\}$:

$$a_C(x_C,t) = \frac{1}{\eta^C_0} + \frac{\beta^C}{\eta^C}\left(\frac{(t - x_C - t^C_0)^+ - \rho_C\left([t] - x_c - t^C_0\right)^+}{\eta^C}\right)^{\beta^C - 1} \tag{11}$$

with $[t]$, the integer part of t, which corresponds to the last preventive maintenance action date occurred before t. Maintenance actions occur every year in the beginning of the year, so when $t \in \mathbf{N}^*$.

When the two components are working, failure of component A in mode F_1 does not cause the system to crash. The failure is not detected so component A is not replaced. PDMP jumps from state $(1,1)$ to state $(0,1)$. The PDMP transition kernel is:

$$b\left(\left((1,1),(x_A,x_B,t)\right),\left((0,1),(dy_A,dy_B,ds)\right)\right) = p^A_{F_1} \cdot a_A(x_A,t)\,\delta_{x_A,x_B,t}(dy_A,dy_B,ds) \tag{12}$$

In the same way, after component B failure in mode F_1, the system is still working and the failure is not detected. The PDMP transition kernel is:

$$b\left(\left(\left(1,1\right),\left(x_A,x_B,t\right)\right),\left(\left(1,0\right),\left(dy_A,dy_B,ds\right)\right)\right)=$$

$$p^B_{F_1}\cdot a_B\left(x_B,t\right)\delta_{x_A,x_B,t}\left(dy_A,dy_B,ds\right) \tag{13}$$

When component A (respectively component B) fails in mode F_2 while component B (respectively component A) is working, the system immediately stops working and the failed component is replaced by a new one. PDMP stays in state $(1,1)$, the transition kernel is:

$$b\left(\left(\left(1,1\right),\left(x_A,x_B,t\right)\right),\left(\left(1,1\right),\left(dy_A,dy_B,ds\right)\right)\right)=$$

$$p^A_{F_2}\cdot a_A\left(x_A,t\right)\delta_{t,x_B,t}\left(dy_A,dy_B,ds\right)$$

$$+\,p^B_{F_2}\cdot a_B\left(x_B,t\right)\delta_{x_A,t,t}\left(dy_A,dy_B,ds\right) \tag{14}$$

When component A is down in mode F_1, a failure of component B leads to a system crash regardless of its failure mode. The two components are replaced by new ones. The PDMP transition kernel is

$$b\left(\left(\left(0,1\right),\left(x_A,x_B,t\right)\right),\left(\left(1,1\right),\left(dy_A,dy_B,ds\right)\right)\right)=$$

$$a_B\left(x_B,t\right)\delta_{t,t,t}\left(dy_A,dy_B,ds\right) \tag{15}$$

It is the same if component A fails while component B is already down in mode F_1:

$$b\left(\left(\left(1,0\right),\left(x_A,x_B,t\right)\right),\left(\left(1,1\right),\left(dy_A,dy_B,ds\right)\right)\right)=$$

$$a_A\left(x_A,t\right)\delta_{t,t,t}\left(dy_A,dy_B,ds\right) \tag{16}$$

Addingly, during a preventive maintenance action, failed components are replaced by new ones. A preventive maintenance action is undertaken in the beginning of the year which means that when $t\in\{1,2,\cdots,[T]\}$. So we define PDMP transition kernels which model replacement of broken components during a preventive maintenance action, for all $k\in\{1,2,\cdots,[T]\}$:

$$q\left(\left(\left(1,1\right),\left(x_A,x_B,k\right)\right),\left(\left(1,1\right),\left(dy_A,dy_B,ds\right)\right)\right)=$$

$$\delta_{x_A,x_B,k}\left(dy_A,dy_B,ds\right) \tag{17}$$

$$q\left(\left(\left(0,1\right),\left(x_A,x_B,k\right)\right),\left(\left(1,1\right),\left(dy_A,dy_B,ds\right)\right)\right)=$$

$$\delta_{k,x_B,k}\left(dy_A,dy_B,ds\right) \tag{18}$$

$$q\left(\left(\left(1,0\right),\left(x_A,x_B,k\right)\right),\left(\left(1,1\right),\left(dy_A,dy_B,ds\right)\right)\right)=$$

$$\delta_{x_A,k,k}\left(dy_A,dy_B,ds\right) \tag{19}$$

In order to initialize the PDMP, we define the initial law $\pi_0(\cdot,dx)$ as the empirical law of entry into service dates of the systems presently working ($t=0$). We suppose that all systems are in state $(1,1)$.

4.2 *PDMP quantification with a finite volume algorithm*

Using the fact that a PDMP is a Markov process (with general state space), the associated Chapman-Kolmogorov equation may be written, see (Eymard et al. 2008). This equation represents some balance in terms of probability flows, which takes into account both of the deterministic evolution between jumps (which evolves with speed $v(i,x)$) and the jumps (governed by $a(i,j,x)\mu_{(i,j,x)}(dy)$ and $q((i,x),(j,dy))$). Finite volume (FV) methods are then known to be well adapted for their numerical resolution. They estimate an approximation of the PDMP marginal distributions denoted by $\left(\pi_t(\cdot,dx)\right)_{t\geq 0}$. Their principle is based on the discretization of both time and environmental state spaces into cells. The mesh of environmental state space is denoted by $\mathcal{M}$. A cell K of mesh $\mathcal{M}$ is written as $K=K_1\times K_2\times K_3$ where K_1, K_2 and K_3 are three intervals. The time evolution of the probability masses in each cell of the environmental state space is followed (time) step by step and at each step, some balance is written between the out- and in-coming probability masses. This brings us to solve a linear system. This method allows us to estimate associated quantities to a maintenance strategy such as mean cost or mean number of failures.

Let $h\in\mathbf{R}^*_+$ be the environmental state space step and $\delta t\in\mathbf{R}^*_+$ the time step. The FV algorithm that we propose here computes an approximation $P^{h,\delta t}_t(i,x)dx$ of $\pi_t(i,dx)$ which admits a density $P^{h,\delta t}_t(i,x)$ with respect of Lebesgue measure, constant on each time step and on each cell K of the environmental state space.

$$P^{h,\delta t}_t(i,x):=u_n(i,K) \tag{20}$$

with $t\in[n\cdot\delta t;(n+1)\cdot\delta t[$.

The discrete transition rate between the cells (i,K) and (j,L) is:

$$a^{i,j}_{K,L}=\frac{1}{h^d}\int_K a\left(i,j,x\right)\int_L \mu_{(i,j,x)}\left(dy\right)dx \tag{21}$$

The discrete exit rate from the cell (i,K) is:

$$b^i_K=\sum_{j\in E}\sum_{L\in\mathcal{M}} a^{i,j}_{K,L} \tag{22}$$

Let $K=K_1\times K_2\times K_3$ be a cell. If it exists $k\in\{1,2,\cdots,[T]\}$ such as $k\in K_3$, the discrete transition rate between the cells (i,K) and (j,L) caused by a deterministic jump is:

$$q^{i,j}_{K,L}(n)=\frac{1}{h^d}\int_K\int_L q((i,x),(j,dy))dx \tag{23}$$

with n such as $k \in [n \cdot \delta t; (n+1) \cdot \delta t[$

If it does not exist $k \in \{1, 2, \cdots, [T]\}$ such as $k \in K_3$ then:

$$q_{K,K}^{i,i}(n) = 1 \text{ and } q_{K,L}^{i,j}(n) = 0 \ \forall (i, K) \neq (j, L) \quad (24)$$

If $K = K_1 \times K_2 \times K_3$ and $L = L_1 \times L_2 \times L_3$ are two neighboring cells of mesh $\mathcal{M}$ such as $K_1 = L_1$ and $K_2 = L_2$ then we set:

- $v_{K,L}^i = 1$ if for all $c \in K_3$ and $d \in L_3$, $d > c$,
- $v_{K,L}^i = -1$ if for all $c \in K_3$ and $d \in L_3$, $d < c$.

In all other cases, we set $v_{K,L}^i = 0$.

The FV algorithm is first initialized by:

$$u_0(i, K) = \frac{1}{h^d} \int_K \pi_0(i, dx) \quad (25)$$

In our case, the FV algorithm writes as follows:

$$\begin{aligned} \widetilde{u}_n(i, K) = {} & u_n(i, K) \\ & - \delta t \left(\sum_{L \in N_K} \frac{1}{h} 1_{v_{K,L}^i > 0} + b_K^i \right) u_n(i, K) \\ & + \delta t \sum_{L \in N_K} \frac{1}{h} 1_{v_{K,L}^i < 0} u_n(i, L) \\ & + \delta t \sum_{j \in E} \sum_{L \in \mathcal{M}} a_{L,K}^{j,i} u_n(j, L) \end{aligned} \quad (26)$$

$$u_{n+1}(i, K) = \sum_{j \in E} \sum_{L \in \mathcal{M}} q_{L,K}^{j,i}(n) \widetilde{u}_n(j, L) \quad (27)$$

A sufficient condition for this algorithm to be stable is that the coefficient of $u_n(i, K)$ is non negative, which may be written:

$$1 - \delta t \left(\sum_{L \in N_K} \frac{1}{h} 1_{v_{K,L}^i > 0} + b_K^i \right) \geq 0 \quad (28)$$

This methodology has already been used to optimize the maintenance of a track circuit (Lair et al. 2009) and of an air conditioning system used in regional train (Lair et al. 2010).

5 MAINTENANCE OPTIMIZATION

5.1 *A new maintenance strategy with preventive renewal of components*

In addition of the preventive maintenance strategy already applied, we propose to replace components based on their age during a preventive maintenance action. If a component is older than an age called 'limit age', it is replaced. To optimize the system maintenance, the components limit ages that minimize the maintenance mean cost should be found. Let r_A be the component A limit age, and r_B be the component B limit age. This new maintenance strategy causes a change for the PDMP transition kernels associated to a preventive maintenance action; for all $k \in \{1, 2, \cdots, [T]\}$:

$$\begin{aligned} & q\left(\left((1,1), (x_A, x_B, k)\right), \left((1,1), (dy_A, dy_B, ds)\right)\right) = \\ & \left(1_{k - x_A < r_A} \delta_{x_A}(dy_A) + 1_{k - x_A \geq r_A} \delta_k(dy_A)\right) \\ & \cdot \left(1_{k - x_B < r_B} \delta_{x_B}(dy_B) + 1_{k - x_B \geq r_B} \delta_k(dy_B)\right) \\ & \cdot \delta_k(ds) \end{aligned} \quad (29)$$

$$\begin{aligned} & q\left(\left((0,1), (x_A, x_B, k)\right), \left((1,1), (dy_A, dy_B, ds)\right)\right) = \\ & \delta_k(dy_A) \cdot \left(1_{k - x_B < r_B} \delta_{x_B}(dy_B) + 1_{k - x_B \geq r_B} \delta_k(dy_B)\right) \\ & \cdot \delta_k(ds) \end{aligned} \quad (30)$$

$$\begin{aligned} & q\left(\left((1,0), (x_A, x_B, k)\right), \left((1,1), (dy_A, dy_B, ds)\right)\right) = \\ & \left(1_{k - x_A < r_A} \delta_{x_A}(dy_A) + 1_{k - x_A \geq r_A} \delta_k(dy_A)\right) \cdot \delta_k(dy_B) \\ & \cdot \delta_k(ds) \end{aligned} \quad (31)$$

We also estimate the impact of doubling the maintenance step on failures number, undesirable event number and maintenance cost.

5.2 *Cost function*

In this article, an optimal maintenance strategy minimizes the maintenance mean cost over T years. The cost function is defined as follows. We consider that a classical failure and the undesirable event are two different events. Let's introduce some notations:

- C_A : component A replacement cost,
- C_B : component B replacement cost,
- C_{un} : undesirable event cost,
- C_f : classical failure cost,
- C_{pm} : preventive maintenance cost,
- $N_{un}(t)$: mean number of undesirable events on $[0; t]$,
- $N_{A,B}^f(t)$: mean number of failures with replacement of components A and B on $[0; t]$,

- $N_A^f(t)$: mean number of failures with replacement of component A only on $[0;t]$,
- $N_B^f(t)$: mean number of failures with replacement of component B only on $[0;t]$,
- $N_A^{pm}(t)$: mean number of component A replacements during a preventive maintenance action on $[0;t]$,
- $N_B^{pm}(t)$: mean number of component B replacements during a preventive maintenance action on $[0;t]$,
- $N_{pm}(t)$: number of preventive maintenance actions on $[0;t]$,

The cost function is given by the following:

$$
\begin{aligned}
C(t) = {} & N_{un}(t) \cdot (C_A + C_B + C_{un}) \\
& + N_{A,B}^f(t) \cdot (C_A + C_B + C_f) \\
& + N_A^f(t) \cdot (C_A + C_f) + N_B^f(t) \cdot (C_B + C_f) \\
& + N_A^{pm}(t) \cdot C_A + N_B^{pm}(t) \cdot C_B \\
& + N_{pm}(t) \cdot C_{pm}
\end{aligned}
\tag{32}
$$

The involved quantities may be expressed in terms of the PDMP marginal distribution. For example, the mean number of undesirable event occurred before time t, $N_{un}(t)$ may be assessed as follows:

$$
\begin{aligned}
& N_{un}(t) = \\
& \int_0^t \int_{[0,T]^3} p_{F_1} a_A(x_A, r) \pi_s((1,0), dx_A, dx_B, dr) ds \\
& + \int_0^t \int_{[0,T]^3} p_{F_1} a_B(x_B, r) \pi_s((0,1), dx_A, dx_B, dr) ds
\end{aligned}
\tag{33}
$$

Thanks to a simulated annealing algorithm, we are able to find components limit ages which minimize cost function with a preventive maintenance step given. Two different preventive maintenance steps are tested : one year (the current one) and two years. Higher values are not tested because they are not allowed by SNCF maintenance rules. The results are given in next section.

5.3 *Results*

Table 1 and Figures 4, 5 and 6 represent comparison between indicators of the current maintenance strategy and three others which are:

Table 1: System cumulative mean quantities over T years for the three maintenance strategies compared to the current one

Preventive maintenance step	1	2	2
Components renewal	yes	no	yes
Cost	-16%	-0.2%	-17%
Undesirable events	-42%	$+200\%$	$+66\%$
Classical failures	-31%	-0.5%	-31%

1. Periodic preventive maintenance with step equal to one (PMS=1) and preventive replacement of components,
2. Periodic preventive maintenance with step equal to two (PMS=2) without preventive replacement of components,
3. Periodic preventive maintenance with step equal to two (PMS=2) and preventive replacement of components.

Let Z be an indicator associated to the current strategy, and $\overline{Z}$ the same indicator but associated to another strategy. Let Δ_Z be a comparison coefficient such as:

$$
\Delta_Z = \left(\frac{\overline{Z}}{Z} - 1 \right) \cdot 100 \tag{34}
$$

We compare three indicators: mean maintenance cost, mean number of undesirable events and mean number of failures. These indicators are estimated for each year of operation (Figures 4, 5 and 6) and are summed over time T (Table 1).

From an economical perspective, the best maintenance strategy corresponds to a maintenance step equal to two and with components renewal, it leads to a cost decrease of -17%. However this strategy leads an increase of the undesirable events number ($+66\%$). Because of that, such strategy can not be applied. Maintenance strategy which corresponds to a maintenance step equal to one and with components renewal leads to a decrease of all quantities (cost -16%, number of undesirable events -42% and number of failures -31%). Such a strategy is a good choice and may be applied. It is interesting to note that doubling the PMS leads to an 200% increase of the mean number of undesirable events but does not increase the mean maintenance cost. This is because an undesirable event is very rare.

In Figure 4, we can observe that doubling the preventive maintenance step does not involve a mean cost decrease. Renewal of components A and B leads to an investment the first period and periodically.

The only strategy which decreases the mean number of undesirable events corresponds to the strategy with a maintenance step equal to one and with renewal of components, see Figure 5.

All tested strategies with components renewal decrease the mean number of failures, see Figure 6.

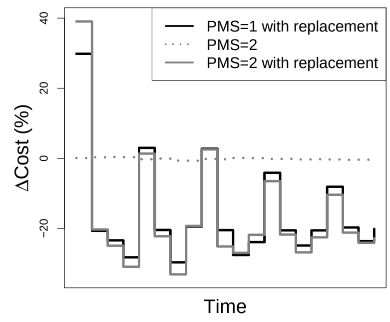

Figure 4: Mean system maintenance cost for the three maintenance strategies compared to the current one

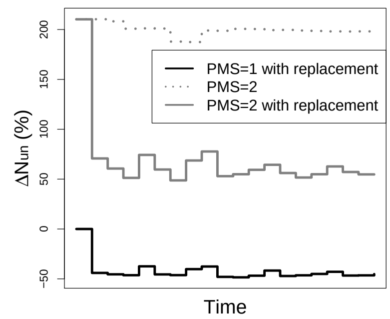

Figure 5: Mean number of system undesirable events for the three different maintenance strategies compared to the current one (N_{un} : number of undesirable events)

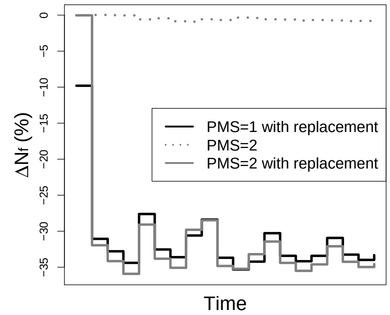

Figure 6: Mean number of system failures for the three different maintenance strategies compared to the current one (N_f : number of failures)

6 CONCLUSIONS

Thanks to an $ARABA_1$ (Arithmetic Reduction of Age with Bertholon Adaptation of order one) model, we have been able to estimate adjustments effect on components life-time jointly with intrinsic failure rate. This model is a modification of the ARA_1 model. It appears that it fits better with our data than an ARA_1 model with Weibull intrinsic failure rate. By taking into account the components life-time distributions, the adjustments effect and the preventive maintenance strategy, we modelled the system with a PDMP. Finite volume method quickly gives us searched quantities which allowed us to test different maintenance strategies. Thus, we found a new preventive maintenance strategy based on components preventive replacement which not only minimizes the maintenance mean cost but reduces both mean number of undesirable events and mean number of failures as well.

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