Optimal replacement policy for components with general failure rates submitted to obsolescence

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ABSTRACT: Identical components are considered, which become obsolete once new-type ones are available, more reliable and less energy consuming. We envision different possible replacement strategies for the old-type components by the new-type ones: purely preventive, purely corrective and different mixtures of both types of strategies. To evaluate the respective value of each possible strategy, a cost function is considered, which takes into account replacement costs, with economical dependence between simultaneous replacements, and energy consumption (and/or production) cost, with a constant rate per unit time. A full analytical expression is provided for the cost function induced by each possible replacement strategy. The optimal strategy is derived in long-time run. Numerical experiments close the paper.

1 INTRODUCTION

Identical and independent components are considered, which may be part of a single industrial equipment or dispatched in different locations, indifferently. Those components are degrading with time and their random life-times follow some common general distribution. At some fixed time, say time 0, new components appear in the market, issued from a new technology, which makes them more reliable, less energy consuming and more performing. Such new-type components may be substituted to the older ones with no problem of compatibility. There is no stocking of old-type components and after time 0, no old-type component is available any more (or the industrialist is not allowed to use old-type components any more, e.g. for safety reasons). After time 0, any failed component, either old-type or new-type, is instantaneously replaced by a new-type one. At time 0, each old-type component is in use since some random time, with some random remaining life-time. If the new-type components are much less energy consuming than the older ones and if the period of interest is very long, it may then be expedient to remove all old-type components immediately at time 0 and replace them by new-type ones, leading to some so-called purely preventive replacement strategy. On the contrary, in case there is not much improvement between both technologies and if the period of interest is short, it may be better to wait until the successive failures of the old-type components and replace them by new-type ones only at failure, leading to some purely corrective replacement strategy. More generally, some mixture of both strategies, preventive and corrective, may also be envisioned (details below) and may lead to lower costs, as will be seen later. The point of the present paper is to look for the optimal replacement strategy with respect of a cost function, which represents the mean total cost on some finite time interval $[0, t]$. This function takes into account replacement costs, with economical dependence between simultaneous replacements (Dekker, Wildeman, and van der Duyn Schouten 1997), and also energy consumption (and/or production) cost, with a constant rate per unit time.

A similar model as here has already been studied in (Elmakis, Levitin, and Lisnianski 2002) and (Mercier and Labeau 2004) in case of constant failures rates for both old-type and new-type components. In those papers, all costs were addingly discounted at time 0, contrary to the present paper. In such a context, it had been proved in (Mercier and Labeau 2004) that in case of constant failure rates, the only possible optimal strategies were either purely corrective or nearly pure preventive (details further), leading to some simple dichotomous decision rule.

A first attempt to see whether such a dichotomy is still valid in case of general failure rates was done in (Michel, Labeau, and Mercier 2004) by Monte-

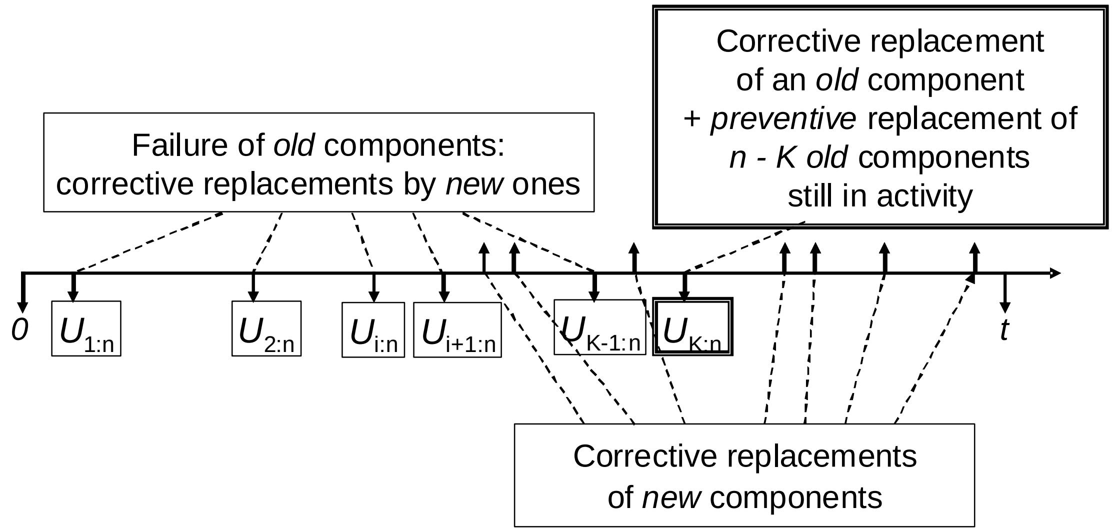

Figure 1: Corrective and preventive replacements

Carlo (MC) simulations. However, the length of the MC simulations did not allow to cover a sufficient range for the different parameters, making the answer difficult. Similarly, recent works (Clavareau and Labeau 2006a) or (Clavareau and Labeau 2006b) e.g. proposed complex models including the present one, which are evaluated by MC simulations. Here again, the length of the MC simulations added to the complexity of the model, do not allow to provide the optimal replacement strategy according to the data of the model.

The point of the present paper hence is to answer to the following questions: is the dichotomy proved in case of constant failure rates still valid in case of general failure rates? If not (and it will not), what are the possible optimal strategies? Finally, how can we find the optimal strategy?

This paper is organized as follows: the model is specified in Section 2. Section 3 presents the theoretical results both for a finite and an infinite time horizon. Numerical experiments are lead on in Section 4. Concluding remarks end the paper in Section 5.

This paper presents the results from (Mercier 2008), with different numerical experiments however. Due to the reduced size of the present paper, no proofs are provided here, which may be found in the quoted paper.

2 THE MODEL

We consider n identical and independent components ($n \geq 2$), called old-type components in the following. At time 0, such old-type components are up, in activity. For each $i = 1, ..., n$, the residual life-time for the $i-$th component is assumed to be some absolutely continuous random variable (r.v.) U_i, where U_i's are not necessarily all identically distributed. The $i-$th (old-type) component is assumed to fail at time U_i. The successive times to failure of the n old-type components are the order statistics of $(U_1, ..., U_n)$. They are denoted by $(U_{1:n}, ..., U_{n:n})$, where $U_{1:n} < ... < U_{n:n}$ almost everywhere (a.e.).

All preventive and corrective replacements (by new-type components) are instantaneous. The following replacement strategies are envisioned:

- strategy 0: the n old-type components are immediately replaced by n new-type ones at time 0. This is a purely preventive strategy. After time 0, there are exactly n new-type components and no old-type component any more,
- strategy 1: no replacement is performed before the first failure, which occurs at time $U_{1:n}$. At time $U_{1:n}$, the failed component is correctively replaced and the $n-1$ non-failed old-type components are simultaneously preventively replaced. This hence is a nearly pure preventive strategy. Before time $U_{1:n}$, there are exactly n old-type components. After time $U_{1:n}$, there are exactly n new-type components,
- strategy K ($1 \leq K \leq n$): no preventive replacement is performed before the $K-$th failure, which occurs at time $U_{K:n}$. This means that only corrective replacements are performed up to time $U_{K:n}$ (at times $U_{1:n}$, ..., $U_{K-1:n}$). At time $U_{K:n}$, the failed component is correctively replaced and the $n-K$ non-failed old-type components are simultaneously preventively replaced. Before

time $U_{1:n}$, there are exactly n old-type components. After time $U_{K:n}$, there are exactly n new-type components. For $K \geq 2$, between times $U_{i:n}$ and $U_{i+1:n}^{-}$ $(1 \leq i \leq K-1)$, there are i new-type components and $n-i$ old-type ones (see Figure 1).

- strategy n: no preventive replacement is performed at all. Before time $U_{1:n}$, there are exactly n old-type components. Between times $U_{i:n}$ and $U_{i+1:n}^{-}$ $(1 \leq i \leq n-1)$, there are i new-type components and $n-i$ old-type ones. After time $U_{n:n}$, there are exactly n new-type components.

Once a new-type component is put into activity at time 0 or at time say $U_{i:n}$, it is next instantaneously replaced at failure by another new-type component. The successive life-times of such components are assumed to form a renewal process with eventual delay $U_{i:n}$; the i.i.d. inter-arrival times are distributed as some non-negative r.v. V with $\mathbb{P}(0 \leq V < \infty) = 1$ and $\mathbb{P}(V > 0) > 0$. The renewal function associated to the non-delayed process is then finite on $\mathbb{R}_+$. Let $\mathbb{E}$ stand for the expectation with respect of the probability measure $\mathbb{P}$ on $(\Omega, \mathcal{A})$ and for $A \subset \mathcal{A}$, let $\mathbf{1}_A$ be the indicator function with $\mathbf{1}_A(\omega) = 1$ if $\omega \in A$ and $\mathbf{1}_A(\omega) = 1$ if $\omega \in \Omega \backslash A$. The renewal function is then denoted by ρ_V with:

$$\rho_V(t) = \mathbb{E}\left(\sum_{k \in \mathbb{N}^*} \mathbf{1}_{\left\{V^{(1)}+\ldots+V^{(k)} \leq t\right\}}\right)$$

for $t \geq 0$, where $V^{(1)}$, ..., $V^{(k)}$, ... are the successive inter-arrival times. We recall that $\rho_V(t)$ corresponds to the mean number of renewals on $[0,t]$ of the non-delayed process.

The envisionned cost function represents the mean total cost on some time interval $[0,t]$. It is denoted by $C_K([0,t])$ when strategy K is used. Two type of costs are considered:

- replacement costs, with economic dependence in case of simultaneous replacements: each solicitation of the repair team is assumed to entail a fixed cost r $(r \geq 0)$. Each corrective and preventive replacement involves a supplementary cost, respectively c_f and c_p, to be added to r $(0 < c_p \leq c_f)$. For instance, the cost for preventive replacement of i units $(0 \leq i \leq n-1)$ which comes along with the corrective replacement of one unit is $r + c_f + ic_p$.

- energy and/or production cost, with a constant rate per unit time (eventually negative, in case of a production rate higher than the energy cost rate). The rates for an old-type and a new type unit respectively are $\eta + \nu$ and η, with $\nu \geq 0$, $\eta \in \mathbb{R}$. (The cost rate is higher for an older unit). The "energy/production" cost for j new-type units and k old-type units on some time intervall $[t_1, t_2]$ is $(j\eta + k(\eta+\nu))(t_2 - t_1)$, where $0 \leq t_1 \leq t_2$ and $j + k = n$.

All components both new-type and old-type are assumed to be independent one with each other.

In all the paper, if X is a non-negative random variable (r.v.), its cumulative density function (c.d.f.) is denoted by F_X, its survival function by $\bar{F}_X$ with $\bar{F}_X = 1 - F_X$ and its eventual probability density function (p.d.f.) by f_X. For $t \in \mathbb{R}_+$, we also set $X^t = \min(X,t)$ and $x^+ = \max(x,0)$ for any real x.

Finally, we shall use the following notations:

$$a = \frac{r + c_f}{c_p} \geq 1$$

$$b = \frac{\nu}{c_p} \geq 0$$

3 THEORETICAL RESULTS

3.1 *Cost functions on* $[0,t]$

We first give our results for a finite mission time t.

Theorem 1 *Let* $t \geq 0$*. For* $K = 0$*, we have:*

$$C_0([0,t]) = n\eta t + r + nc_p(1 + a\rho_V(t))$$

and, for $1 \leq K \leq n$*, we have:*

$$\begin{aligned} C_K([0,t]) &= \sum_{i=1}^{K} \left[(r+c_f)\left(F_{U_{i:n}}(t) + \mathbb{E}\left(\rho_V\left((t-U_{i:n})^+\right)\right)\right) + \nu\mathbb{E}\left(U_{i:n}^t\right)\right] \\ &+ (n-K)\left[(r+c_f)\mathbb{E}\left(\rho_V\left((t-U_{K:n})^+\right)\right) + c_p F_{U_{K:n}}(t) + \nu\mathbb{E}\left(U_{K:n}^t\right)\right] \\ &+ n\eta t \end{aligned}$$

Setting

$$g_K(t) := \frac{1}{c_p}\left(C_{K+1}([0,t]) - C_K([0,t])\right) \quad (1)$$

for all $0 \leq K \leq n-1$, we easily derive the following corollary.

Corollary 2 *Let* $t \geq 0$*. For* $K = 0$*, we have:*

$$\begin{aligned} g_0(t) &= (a-1)F_{U_{1:n}}(t) - \frac{r}{c_p} \\ &+ n\left[b\mathbb{E}\left(U_{1:n}^t\right) - \bar{F}_{U_{1:n}}(t) - a\mathbb{E}\left(\rho_V(t) - \rho_V\left((t-U_{1:n})^+\right)\right)\right] \end{aligned}$$

and, for $1 \leq K \leq n-1$, we have:

$$g_K(t)$$

$$= (a-1) F_{U_{K+1:n}}(t)$$

$$+ (n-K) \times \left[b\mathbb{E}\left(U^t_{K+1:n} - U^t_{K:n}\right) \right.$$
$$- \left(F_{U_{K:n}}(t) - F_{U_{K+1:n}}(t)\right)$$
$$\left. -a\mathbb{E}\left(\rho_V\left((t-U_{K:n})^+ - \rho_V\left((t-U_{K+1:n})^+\right)\right)\right)\right]$$

In order to find the optimal strategy according to the mission time t and to the data of the model as in the case of constant failure rates (see (Mercier and Labeau 2004)), the point should now be to find out the sign of $g_K(t)$ for $0 \leq k \leq n-1$. This actually seems to be impossible in the most general case. However, we are able to give some results in long-time run, which is done in next subsection.

3.2 *Comparison between strategies 0, 1, ..., n in long-time run*

We first compute the limit of $g_K(t)$ when $t \to +\infty$.

Proposition 3 *Assume the distribution of V to be nonarithmetic and $\mathbb{E}(U_i) < +\infty$ for all $1 \leq i \leq n$. Setting $g_K(\infty) := \lim_{t\to+\infty} g_K(t)$ for all $0 \leq K \leq n-1$, we then have:*

$$g_K(\infty)$$

$$= a - 1 + \left(b - \frac{a}{\mathbb{E}(V)}\right)(n-K)\,\mathbb{E}(U_{K+1:n} - U_{K:n})$$

for all $1 \leq K \leq n-1$ and

$$g_0(\infty) = \frac{c_f}{c_p} - 1 + \left(b - \frac{a}{\mathbb{E}(V)}\right) n\mathbb{E}(U_{1:n} - U_{0:n})$$

where we set $U_{0:n} := 0$.

A first consequence is that, if $b - \frac{a}{\mathbb{E}(V)} \geq 0$ or alternatively $\nu \geq \frac{r+c_f}{\mathbb{E}(V)}$, we then have $g_K(\infty) \geq 0$ for all $0 \leq K \leq n-1$ (we recall that $a \geq 1$ and $c_f \geq c_p$). Consequently, if $\nu \geq \frac{r+c_f}{\mathbb{E}(V)}$, the best strategy among $0, ..., n$ in long-time run is strategy 0. Such a result is conform to intuition: indeed, let us recall that ν stands for the additional energy consumption rate for the old-type units compared to the new-type ones; also, observe that $\frac{r+cf}{\mathbb{E}(V)}$ is the cost rate per unit time for replacements due to failures among new-type components in long-time run. Then, the result means that if replacements of new-type components due to failures are less costly per unit time than the benefit due to a lower consumption rate, it is better to replace old-type components by new-type ones as soon as possible.

Now, we have to look at the case $b - \frac{a}{\mathbb{E}(V)} < 0$ and for that, we have to know something about the monotony of

$$D_K := (n-K)(U_{K+1:n} - U_{K:n}),$$

with respect of K, where D_K is the K-th normalized spacing of the order statistics $(U_{1:n}, ..., U_{n:n})$, see (Barlow and Proschan 1966) or (Ebrahimi and Spizzichino 1997) e.g.. With that aim, we have to put some assumption on the distributions of the residual life times of the old-type components at time $t = 0$ (U_i for $1 \leq i \leq n$): following (Barlow and Proschan 1966), we assume that U_1, ..., U_n are i.i.d. IFR (Increasing Failure Rate), which implies that $(D_K)_{0\leq K\leq n-1}$ is stochastically decreasing. A first way to meet with this assumption is to assume that all old-type components have been put into activity simultaneously (before time 0) so that the residual life times are i.i.d. (moreover assumed IFR). Another possibility is to assume that all units have already been replaced a large number of times. Assuming such replacement times for the i-th unit to make a renewal process with inter-arrival times distributed as some $U^{(0)}$ (independent of i), the residual life at time 0 for the i-th unit may then be considered as the waiting time until next arrival for a stationary renewal process with inter-arrivals distributed as $U^{(0)}$. Such a waiting time is known to admit as p.d.f. the function $f_U(t)$ such that:

$$f_U(t) = \frac{\bar{F}_{U^{(0)}}(t)}{\mathbb{E}\left(U^{(0)}\right)} \mathbf{1}_{\mathbb{R}^+}(t), \tag{2}$$

assuming $0 < \mathbb{E}\left(U^{(0)}\right) < +\infty$. Also, it is proved in (Mercier 2008) that if $U^{(0)}$ is IFR, then U is IFR too. The r.v. U_1, ..., U_n then are i.i.d. IFR, consequently meeting with the required assumptions from (Barlow and Proschan 1966).

We are now ready to state our main result:

Theorem 4 *If $b - \frac{a}{\mathbb{E}(V)} \geq 0$, the optimal strategy among $0, ..., n$ in long time-run is strategy 0.*

In case $b - \frac{a}{\mathbb{E}(V)} < 0$, assume that U_1, ..., U_n are i.i.d. IFR r.v. (which may be realized through assuming that U_i stands for the waiting time till next arrival for a stationary renewal process with inter-arrival time distributed as $U^{(0)}$, where $U^{(0)}$ is a non-negative IFR r.v. with $0 < \mathbb{E}\left(U^{(0)}\right) < +\infty$). Assume too that U_i's are not exponentially distributed. The sequence $(\mathbb{E}(D_K))_{0\leq K\leq n-1}$ is then strictly decreasing, and, setting

$$c := \frac{a-1}{\frac{a}{\mathbb{E}(V)} - b} \text{ and } d := \frac{\frac{c_f}{c_p} - 1}{\frac{a}{\mathbb{E}(V)} - b} \leq c,$$

one of the following cases occurs:

- *if* $c \leq \mathbb{E}(D_{n-1})$: *the optimal strategy among* $0, ..., n$ *in long time-run is strategy n,*
- *if* $c > \mathbb{E}(D_1)$:
  - *if* $d > \mathbb{E}(D_0)$: *the optimal strategy among* $0, ..., n$ *in long time-run is strategy 0,*
  - *if* $d \leq \mathbb{E}(D_0)$: *the optimal strategy among* $0, ..., n$ *in long time-run is strategy 1,*
- *if* $\mathbb{E}(D_{K_0}) < c \leq \mathbb{E}(D_{K_0-1})$ *for some* $2 \leq K_0 \leq n-1$: *the optimal strategy among* $0, ..., n$ *in long time-run is strategy* K_0.

In (Mercier and Labeau 2004), we had proved the following "dichotomy" property: in case of constant failure rates, only purely preventive (0), nearly pure preventive (1) or purely corrective (n) strategies can be optimal for finite horizon. We now know from last point of Theorem 4 that such a property is not valid any more in case of general failure rates, at least for infinite horizon and consequently for large t. We now look at some numerical experiments to check the validity of the dichotomy property in case of small t.

4 NUMERICAL EXPERIMENTS

We here assume that U_i's are i.i.d IFR random variables with known distribution. Examples are provided in (Mercier 2008) for the case where the data is the distribution of some $U^{(0)}$ and the common p.d.f. f_U of U_i is given by (2) (see Theorem 4). All the computations are made with Matlab.

All U_i's and V_i's are Weibull distributed according to $W(\alpha_U, \beta_U)$ and $W(\alpha_V, \beta_V)$, respectively, (all independent) with survival functions:

$$\bar{F}_U(x) = e^{-\alpha_U x^{\beta_U}} \text{ and } \bar{F}_V(x) = e^{-\alpha_V x^{\beta_V}}$$

for all $x \geq 0$.

We take:

$$\alpha_U = 1/10^3; \alpha_V = 1/\left(2.25 \times 10^3\right) \quad (3)$$

$$\beta_U = \beta_V = 2.8 > 1 \quad (4)$$

(U_i's are IFR), which leads to

$$\mathbb{E}(U) \simeq 10.5, \sigma(U) \simeq 4.1,$$

$$\mathbb{E}(V) \simeq 14, \sigma(V) \simeq 5.4.$$

We also take:

$$n = 10; \eta = 0; \nu = 0.06; c_p = 1; c_f = 1.1; r = 0 \quad (5)$$

We compute $F_{U_{K:n}}$ using:

$$\begin{aligned} &F_{U_{K:n}}(x) \\ &= \int_0^{F_U(x)} \frac{n!}{(K-1)!\,(n-K)!} t^{K-1} (1-t)^{n-K}\, dt \\ &= I_{F_U(x)}(K, n-K+1) \end{aligned}$$

for $1 \leq K \leq n$, where $I_x(n_1, n_2)$ is the incomplete Beta function (implemented in Matlab), see (Arnold, Balakrishnan, and Nagaraja 1992) e.g. for the results about order statistics used in this section.

We also use:

$$\bar{F}_{U_{K+1:n}}(t) - \bar{F}_{U_{K:n}}(t) = \binom{n}{K} F_U^K(t)\, \bar{F}_U^{n-K}(t)$$

from where we derive $\mathbb{E}\left(U^t_{K+1:n} - U^t_{K:n}\right)$ due to:

$$\begin{aligned} &\mathbb{E}\left(U^t_{K+1:n} - U^t_{K:n}\right) \\ &= \int_0^t \left(\bar{F}_{U_{K+1:n}}(u) - \bar{F}_{U_{K:n}}(u)\right) du \end{aligned}$$

for $0 \leq K \leq n-1$ (we recall $U_{0:n} := 0$).

Table 1. Optimal strategy according to t and α_V.

$t \setminus \frac{1}{\alpha_V}\,(\times 10^3)$	1	1.2	1.5	1.75	2	2.25	2.5	3	3.5	4	5
5	10	10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10	10	10
15	10	10	10	2	2	1	1	1	0	0	0
20	7	7	6	5	5	4	4	3	2	1	1
25	9	9	8	8	7	7	6	5	4	3	1
30	10	10	9	9	8	7	6	4	3	1	0
35	9	9	7	6	5	4	4	2	1	0	0
40	10	9	8	7	6	5	4	3	2	1	0
45	10	9	8	7	6	6	5	3	2	1	0
50	10	9	8	7	6	5	4	3	2	1	0
75	10	9	8	7	6	5	4	3	2	1	0
100	10	9	8	7	6	5	4	3	2	1	0
∞	10	9	8	7	6	5	4	3	2	1	0

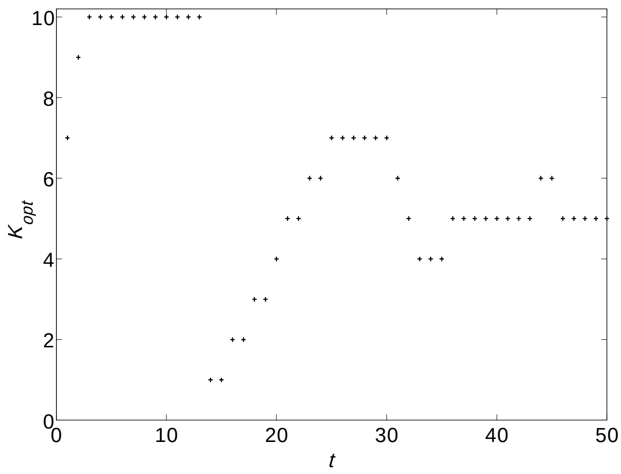

Figure 2: Optimal strategy with respect of t

We finally compute $\mathbb{E}\left(\rho_V\left((t-U_{K:n})^+\right)\right)$ with:

$$\mathbb{E}\left(\rho_V\left((t-U_{K:n})^+\right)\right)$$

$$= \int_0^t \rho_V(t-u)\, df_{U_{K:n}}(t)$$

$$= n\binom{n-1}{K-1}$$

$$\times \int_0^t \rho_V(t-u)\, F_U^{K-1}(u)\, \bar{F}_U^{n-K}(u)\, f_U(u)\, du$$

where the renewal function ρ_V is computed via the algorithm from (Mercier 2007).

For finite horizon, the optimization on K is simply made by computing all $C_K([0,t])$ for $K = 0, ..., n$ and taking the smallest. For infinite horizon, Theorem 4 is used.

The optimal strategy is given in Table 1 for different values of α_V and t, as well as the asymptotic results (all other parameters fixed according to $(3-5)$. We can see in such a table that the optimal strategy is quickly stable with increasing t. More precisely, the optimal strategy for a finite horizon t is the same as the optimal strategy in long-time run as soon as t is greater than about 3.5 mean lengths of life of a new-type component. For t about twice the mean life length, the finite time optimal strategy is already very near from the long-time run one. Also, any strategy may be optimal, even for small t.

We now plot in Figure 2 the optimal strategy with respect of t, for α_V fixed according to (3). We can see in such a figure that the behavior of K_{opt} (optimal K) with increasing t is not regular at all. There is consequently no hope to get any clear characterization of K_{opt} with respect of the different parameters in finite horizon as we had in the exponential case in (Mercier and Labeau 2004) and as we have here in infinite horizon (Theorem 4).

We next plot K_{opt} in Figures 3-6 for t fixed ($t = 25$) with respect of parameters c_f, ν, r and c_p (all other parameters fixed according to $(3-5)$), which shows that K_{opt} may vary a lot changing one single parameter. Also, one may note that K_{opt} decreases with c_f (Fig. 3), ν (Fig. 4) and r (Fig. 5). Such observations are coherent with intuition which says that preventive maintenance should be performed all the earlier (or equivalently new-type components should be introduced all the earlier) as failures are more costly, as the difference of costs is higher between both generations of components, or as economical dependance between replacements is higher. Similarly, Figure 6 shows that K_{opt} increases with c_p, which means that the higher the cost of a preventive replacement is, the later the preventive maintenance must be performed. This is coherent with intuition, too.

5 CONCLUSION

In conclusion, we have considered here different replacement strategies for obsolescent components by others issued from a newer technology. A cost function on a finite and on an infinite time horizon has been considered, in order to sort the different strategies one with each other. We have seen that the variations of the optimal strategy with respect of a finite horizon t is much less regular in the present case of general failure rates than in the case of constant failure rates as in (Elmakis, Levitin, and Lisnianski 2002) or (Mercier and Labeau 2004) (see Figure 2). Also, the main result from (Mercier and Labeau 2004), which told that the optimal strategy could only be strategy 0, 1 or n, namely (nearly) purely preventive or purely corrective, is here false: any strategy among 0, 1, ..., n may be optimal.

It does not seem possible here to give clear conditions on the data to foretell which strategy is optimal in finite horizon as in case of constant failure rates. We however obtained such conditions in long-time run. A few numerical experiments (see others in (Mercier 2008)) seem to indicate that the optimal strategy in long-time run actually is quickly optimal, namely for t not that large. The results for long-time run then seem to give a good indicator for the choice of the best strategy, even for t not very large.

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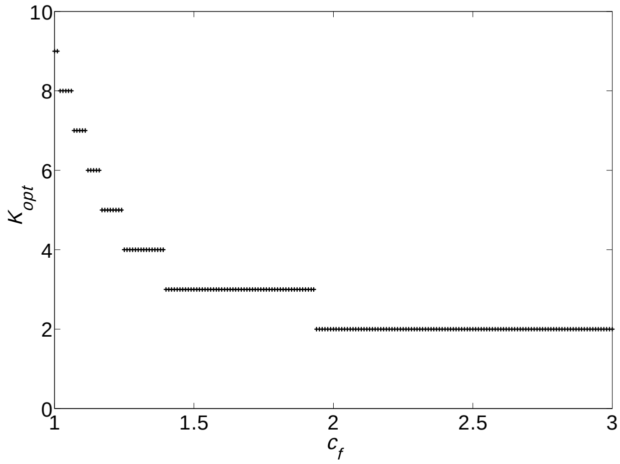

Figure 3: Optimal strategy w. r. of c_f for $t = 25$

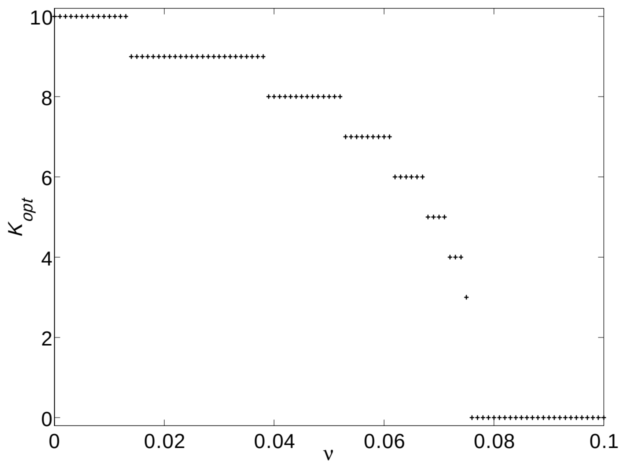

Figure 4: Optimal strategy w. r. of ν for $t = 25$

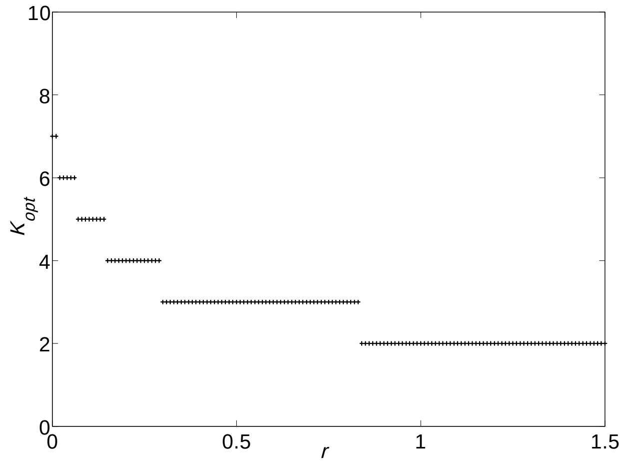

Figure 5: Optimal strategy w. r. of r for $t = 25$

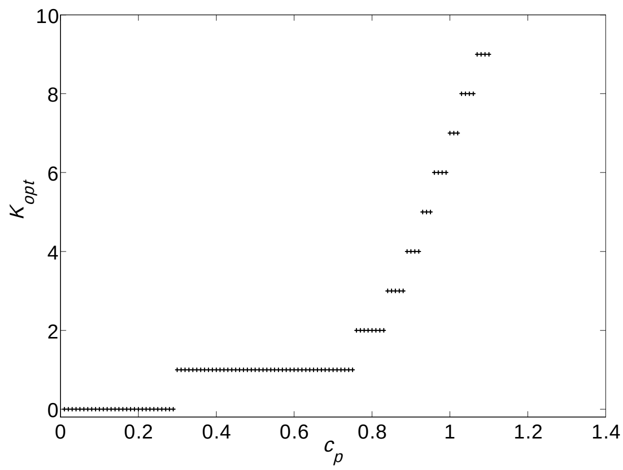

Figure 6: Optimal strategy w. r. of c_p for $t = 25$

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